Lectures 1-2: Time Value of Money

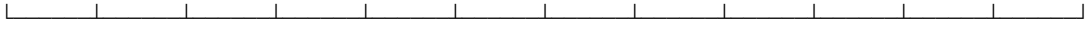
I. Reading
   A. RWJ Chapter 5.

II. Time Line
   A. $1 received today is not the same as a $1 received in one period's time; the timing of a cash flow affects its value.
   B. Hence, when valuing cash flow streams, the timing of the cash flows is crucial: a good idea is to draw a time line.



$100

is not the same as

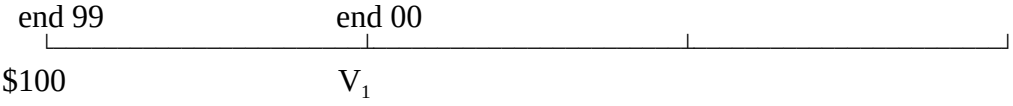


$100

III. Interest Rate: Discrete Compounding

A. Example:

1. Question: Today is the start of 2000. Suppose I can invest $100 at an effective annual interest rate of 12%. What is my $100 worth at the end of the year?



2. Answer: V_0=$100; Interest = $100 x 0.12 = $12
V_1= V_0 + Interest = $100+$12 = $100(1+0.12)=$112.



B. Definition:

1. The effective interest rate r (expressed as a decimal) over any period tells what $x will be worth at the end of the period using the following formula: $x (1+r).
2. The effective interest rate r (expressed as a decimal) over any period from t to (t+1) satisfies:

$$V_{t+1} = V_t (1+r)$$

where V_t is the value at time t and V_{t+1} is the value at time t+1.

C. Example:

1. Question: Today is the start of 2000. Suppose I can invest $100 at the start of 2001 at an effective annual interest rate of 12%. What is my $100 worth at the end of 2001?

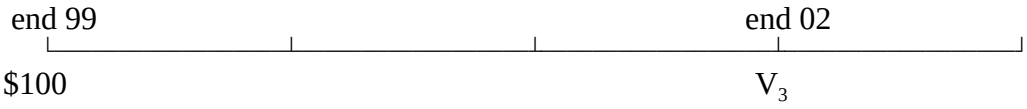


2. Answer: V_1=$100; V_2= V_1(1+0.12)=$100(1+0.12)=$112.

IV. Single Sums: Multiple Periods and Future Values

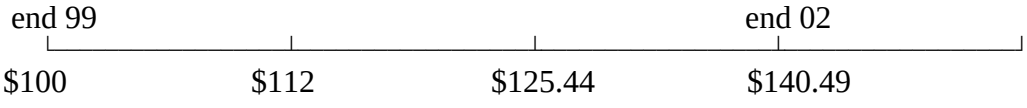
A. Example (cont):

1. Question: Today is the start of 2000. Suppose I can invest $100 at an effective annual interest rate of 12%. What is my $100 worth at the end of 3 years?



2. One Answer: Obtain V_3 in three steps

V_0=$100
$V_1=V_0 (1+r)$=$100(1+0.12)$=$112
$V_2=V_1 (1+r)$=$112(1+0.12)$=$125.44
$V_3=V_2 (1+r)$=$125.44(1+0.12)$=$140.49



3. Another Answer: Can see that the 3 steps could be combined into 1

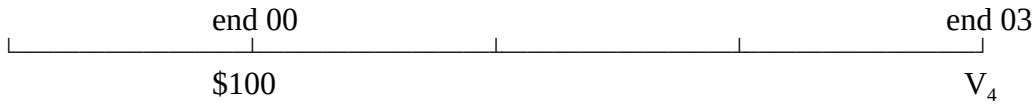
V_0=$100
$V_3=V_0 (1+r)(1+r)(1+r)$=$100(1+0.12)(1+0.12)(1+0.12)$=$140.49
$V_3=V_0 (1+r)^3$=$100(1+0.12)^3$=$140.49

4. Notice how the answer is the initial investment V_0 times a multiplier that only depends on the effective interest rate and the investment interval. This multiplier is known as the future value interest factor.

B. The future value formula answers the following question
   1. If we invest some money at a given effective interest rate, how much money would we have at some future time?
   2. If we invest V_0 today at a given effective interest rate per period of r (expressed as a decimal), how much money would we have in n periods time; i.e., what is V_n?
   3. If we invest V_t in t periods time at a given effective interest rate per period of r (expressed as a decimal), how much money would we have after n periods from investing the money; i.e., what is V_{t+n}?

C. The general future value formula:
   1. $V_{t+n} = V_t (1+r)^n$ where $[(1+r)^n] = FVIF_{r,n}$ is the future value interest factor.
   2. Notice that the future value interest factor does not depend on when the money is invested.

D. Example:
   1. Question: Today is the start of 2000. Suppose I can invest $100 at the end of 2000 at an effective annual interest rate of 12%. What is my $100 worth after being invested for 3 years?



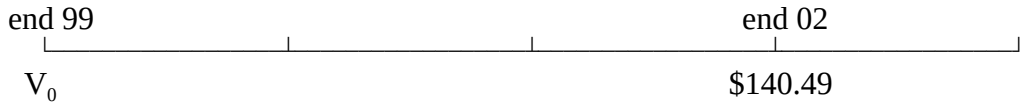
   2. Answer: Use the future value formula. V_1=$100

$V_4=V_1 \times FVIF_{0.12,3} = V_1 (1+0.12)^3=\$100(1+0.12)^3=\$140.49$

V. Single Sums: Multiple Periods and Present Values

A. Example cont:

1. Question: Today is the start of 2000. Suppose I can invest at an effective annual interest rate of 12%. How much do I need to invest today to have $140.49 at the end of 3 years?

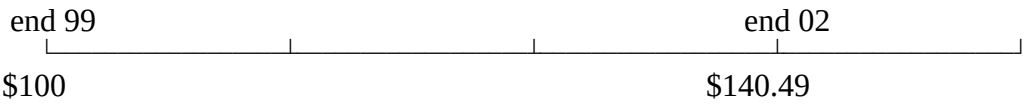


2. Answer: Use future value formula which tells you (from above)

$V_3=V_0 (1+r)^3=\$100(1+0.12)^3=\$140.49$

which implies

$V_0=V_3 /(1+r)^3=\$140.49/(1+0.12)^3=\$100$

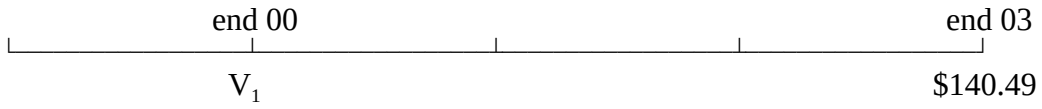


3. Notice how the answer is the final value V_3 times a multiplier that only depends on the effective interest rate and the investment interval. This multiplier is known as the present value interest factor.

B. The present value formula answers the following question
   1. If we can invest money at a given effective interest rate, how much money do we need to invest today to have a given sum at some future time?
   2. If we can invest money at a given effective interest rate r (expressed as a decimal), how much money do we need to invest today V_0 to have a given sum V_n in n periods time?
   3. If we can invest money at a given effective interest rate r (expressed as a decimal), how much money do we need to invest in t periods time V_t to have a given sum V_{t+n} in (t+n) periods from today?

C. The general present value formula:
   1. $V_t = V_{t+n} [1/(1+r)^n]$ where $[1/(1+r)^n]=[(1+r)^{-n}] = PVIF_{r,n}$ is the present value interest factor.

D. Example:
   1. Question: Today is the start of 2000. Suppose I can invest at an effective annual interest rate of 12%. How much do I need to invest at the start of 2001 to have $140.49 at the end of 4 years from today?



   2. Answer: Use the present value formula:

$V_1=V_4 \times PVIF_{0.12,3} = V_4 /(1+r)^3=\$140.49/(1+0.12)^3=\$100$

VI. Equivalent Effective Interest Rates Over Different Compounding Periods.

A. Example:

1. Question: Suppose the effective annual interest rate is 12%. What is the effective 3 year interest rate?
2. Answer: Showed above that

$V_3=V_0\,(1+r)^3 = \$100(1+0.12)^3 = \$100(1+0.4049) = \$140.49$

which implies using the definition of effective rate that the effective 3-year rate is 40.49%

3. Note that Effective 3-year rate = 40.49% $\neq$ 3 x 12% = 3 x Effective 1-year rate.

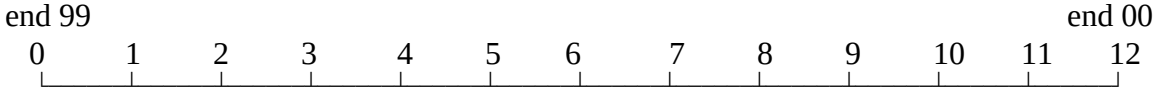
B. General relation between the effective rates for compounding periods of different lengths:

1. The effective n-period rate r_n (expressed as a decimal) is related to the effective one period rate

$(1+r_n) = (1+r)^n$.

C. Example

1. Question: Suppose the effective monthly rate is 0.94888%. What is the effective annual rate?



2. Answer: Here one period is a month. r=0.0094888. Using the effective rate formula

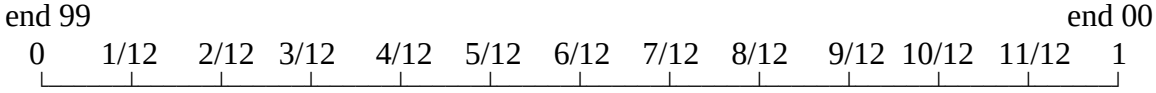
$1+r_{12} = (1+r)^{12}=1.0094888^{12} =1.12$

and so $r_{12}=0.12$ and the effective annual rate is 12%.

D. Does the effective rate formula apply for n a fraction; e.g., if the effective 1-year rate is known, can the monthly effective rate be calculated by using the formula with n = 1/12? The answer is yes.

E. Example

1. Question: Suppose the effective annual rate is 12% What is the effective monthly rate?



2. Answer: Here one period is a year. r=0.12. Using the effective rate formula

$1+r_{1/12} = (1+r)^{1/12}=1.12^{1/12} =1.009488$

and so $r_{1/12}$=0.009488 and the effective monthly rate is 0.9488%.

F. EAR

1. Definition: EAR is the effective annual rate.

VII. Alternate Interest Rate Concepts

A. Nominal Rate or APR

1. Definition: when the compounding period is some fraction of a year 1/m, the nominal rate i_{nom} (expressed as a decimal) equals $m\ r_{1/m}$ where $r_{1/m}$ is the effective 1/m-year rate (expressed as a decimal).
2. Example: if the compound period is a month (m=12) and the effective monthly rate is 0.94888% then $r_{1/12}$=0.0094888, i_{nom} = 12 x 0.0094888=0.11387 and the nominal rate is 11.387%.
3. Fact: the nominal rate only equals the effective annual rate when the compound period is a year.
4. Example (cont): with a compound period of a month, the nominal rate is 11.387% while the effective annual rate is 12%.
5. Periodic rate
   a. Definition: when the compounding period is some fraction of a year 1/m, the periodic rate (expressed as a decimal) equals the effective 1/m-year rate $r_{1/m}$.

B. Continuous Compounding.

1. Definition:

a. Let r be the effective interest rate for one period and define r', the continuous interest rate for a period as follows:

$e^{r'} = (1+r)$.

b. Thus, the continuous rate can be obtained from the effective rate as follows:

r' = ln (1+r).

2. Example: The effective annual rate is 12%.

a. Question: What is the continuously compounded annual rate?

b. Answer: r=0.12. The continuously compounded annual rate (expressed as a decimal) is r' where r'= ln(1+r) =ln(1.12)=0.1133.

3. Rules for using continuous interest rates.

a. The continuous interest rate for any fraction of a period n is given by $r'_n = nr'$ (c.f. effective interest rates for which this is not true $r_n \neq nr$). This additivity is a major advantage of continuous compounding.

b. To obtain future values, use the following formula:

$V_{t+n} = V_t \exp\{nr'\}$.

c. To obtain present values, use the following formula:

$V_t = V_{t+n} \exp\{-nr'\}$.

4. Example (cont): The effective annual rate is 12%.

a. Question: What is the continuously compounded semiannual rate?

b. Answer: The continuously compounded annual rate is 11.33%; i.e., r' = 0.1133. So the continuously compounded semiannual rate is (11.33/2)%=5.666% since $r'_{1/2} = (1/2)\, r' = 0.5 \times 0.1133 = 0.05666$.

c. Question: How much will $500 invested today be worth in 6 months?

d. Answer: Two approaches to obtaining what $500 will be worth in 6 months:

(1) Continuous Compounding: $V_{1/2} = \$500 \exp\{(1/2)\, r'\} = \$500 \exp\{0.05666\} = \$500 \times 1.0583005 = \$529.15$.

(2) Discrete Compounding: $V_{1/2} = \$500\,(1+r)^{1/2} = \$500 \times 1.12^{1/2} = \$500 \times 1.0583005 = \$529.15$.

C. Interpretation of the continuously compounded interest rate:

1. It can be shown that $e^{r'} = \lim_{m \to \infty} [\ (1 + \frac{r'}{m})^m\ ]$.

2. So r' can be interpreted as the nominal interest rate associated with an infinitesimally small compound period.

D. Formulas.

1. From Nominal Rate to EAR (both expressed as decimals)

$$r = (1 + \frac{i_{nom}}{m})^m - 1$$

2. From EAR to Nominal Rate (both expressed as decimals)

$$i_{nom} = [(1+r)^{1/m} - 1]\ m$$

Example:

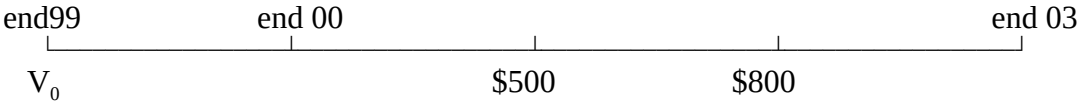
Compound Period	EAR	Nominal Rate	EAR	Nominal Rate
1 year	12%	12%	15%	15%
6 months	12%	11.6601%	15.5625%	15%
3 months	12%	11.4949%	15.8650%	15%
1 month	12%	11.3866%	16.0755%	15%
1 day (365 days=1year)	12%	11.3346%	16.1798%	15%
continuous	12%	11.3329%	16.1834%	15%

VIII. Multiple Cash Flows.

A. Example:

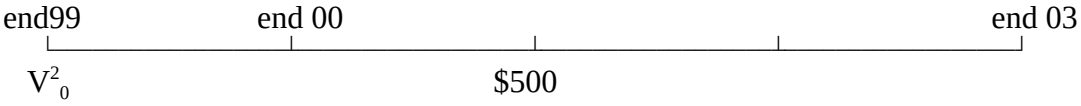
1. Question: Today is the start of 2000. i. How much do I need to invest today at an effective annual rate of 10% to meet a $500 obligation in 2 years and a $800 payment in 3 years? ii. How much would I have to invest if I delay my investment date 1 year?

i.



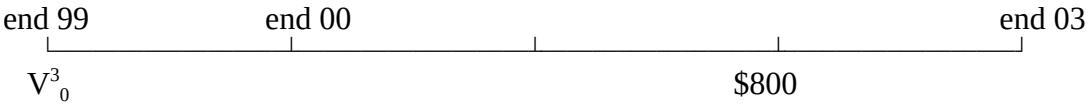
2. Answer i.:

a. using present value formula, amount needed to be invested today to meet the $500 obligation in 2 years:



$V^2_0 = \$500\ PVIF_{0.1,2} = \$500\ (1+0.1)^{-2} = \$413.22$

b. using present value formula, amount needed to be invested today to meet the $800 obligation in 3 years:



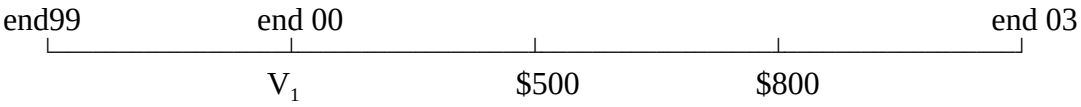
$V^3_0 = \$800\ PVIF_{0.1,3} = \$800\ (1+0.1)^{-3} = \$601.05$

c. so total amount needed to be invested today is

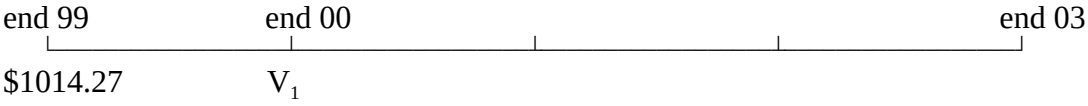
$V_0 = V^2_0 + V^3_0 = \$413.22 + \$601.05 = \$1014.27.$

3. Question: ii. How much would I have to invest if I delay my investment date 1 year?

ii.



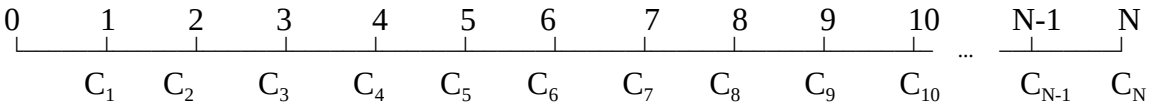
4. Answer ii.:
   a. once the stream of cash flows has been converted to a single sum at a certain point in time (here time 0), can use the present and future value formulas to convert the stream to a single sum at any other time.
   b. so total amount that would have to be invested in one year using the future value formula:



V_1= \$1014.27 $FVIF_{0.1,1}$ = \$1014.27 (1+0.1) = \$1115.70

B. Rules

1. Once each of a set of cash flows at different points in time has been converted to a cash flow at the same point in time, those cash flows can be added to get the value of the set of cash flows at that point.



$V^1_0 = C_1\ PVIF_{r,1} = C_1\ (1+r)^{-1}$
$V^2_0 = C_2\ PVIF_{r,2} = C_2\ (1+r)^{-2}$
$V^3_0 = C_3\ PVIF_{r,3} = C_3\ (1+r)^{-3}$
$V^4_0 = C_4\ PVIF_{r,4} = C_4\ (1+r)^{-4}$
...
$V^{N-1}_0 = C_{N-1}\ PVIF_{r,N-1} = C_{N-1}\ (1+r)^{-(N-1)}$
$V^N_0 = C_N\ PVIF_{r,N} = C_N\ (1+r)^{-N}$

———

$V_0 = V^1_0 + V^2_0 + V^3_0 + V^4_0 + ... + V^{N-1}_0 + V^N_0$

2. The value of the stream of cash flows at any other point can then be ascertained using the present or future value formulas:

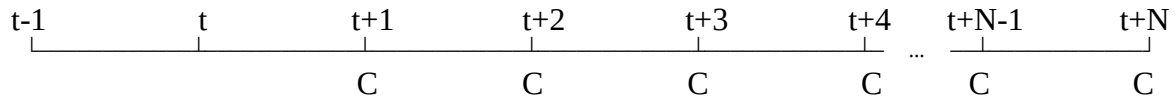
$V_n = V_0\ FVIF_{r,n} = V_0\ (1+r)^n$.

3. So cash flows occurring at different points in time can not be added but cash flows which occur at the same time can be added.

IX. Particular Cash Flow Pattern: Annuity

A. Definition:

1. N equal payments made at equal intervals (which can be more or less than one period).

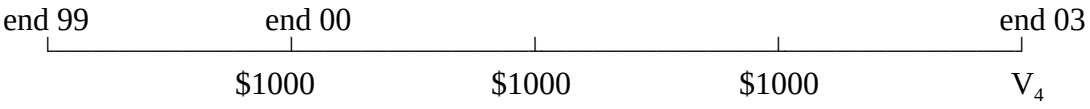


B. Converting to a single sum:

1. One Approach: convert each cash flow to a single sum at a given point in time using the present or future value formula and then add up these sums to give the annuity's value at that point in time.
2. Another Approach:
   a. choose the period so that it is equal to the interval between cash flows.
   b. the annuity's first cash flow occurs at (t+1) and its last at (t+N).
   c. use the present value annuity factor ($PVAF_{r,N}$) which satisfies $V_t = C \times PVAF_{r,N}$ where
      (1) the effective interest rate over a period is r.
      (2) the formula gives the single sum equivalent at the point in time one period before the first cash flow.
      (3) $$PVAF_{r,N} = \frac{1 - (1+r)^{-N}}{r} \quad .$$
   d. or use the future value annuity factor ($FVAF_{r,N}$) which satisfies $V_{t+N} = C \times FVAF_{r,N}$ where
      (1) the effective interest rate over a period is r.
      (2) the formula gives the single sum equivalent at the point in time corresponding to the last cash flow.
      (3) $$FVAF_{r,N} = \frac{(1+r)^{N} - 1}{r} \quad .$$
   e. notice that
      (1) $FVAF_{r,N} = PVAF_{r,N} (1+r)^{N}$; and so
      (2) $V_{t+N} = V_t (1+r)^{N}$ which is consistent with the future value formula.

C. Example:

1. Question: Today is the start of 2000. Suppose I receive \$1000 at the end of each year for the next 3 years. If I can invest at an effective annual rate of 10%, how much would I have in 4 years time?



2. One Answer: using the future value formula

$V^1_4 = \$1000\ FVIF_{0.1,3} = \$1000\ (1+0.1)^3 = \$1331$
$V^2_4 = \$1000\ FVIF_{0.1,2} = \$1000\ (1+0.1)^2 = \$1210$
$V^3_4 = \$1000\ FVIF_{0.1,1} = \$1000\ (1+0.1)^1 = \$1100$

$V_4 = V^1_4 + V^2_4 + V^3_4 = \$1331 + \$1210 + \$1100 = \$3641$

3. Another Answer: using the future value annuity formula gives V_3 and then can use the future value formula to get V_4

$V_3 = C \times FVAF_{0.1,3} = \$1000\ [\{(1+0.1)^3-1\}/0.1] = \$3310$

$V_4 = V_3\ FVIF_{0.1,1} = V_3\ (1+0.1) = \$3310 \times 1.1 = \$3641.$

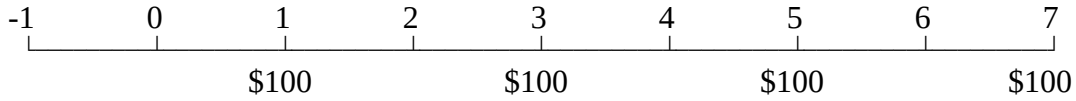
4. Another Answer: using the present value annuity formula gives V_0 and then can use the future value formula to get V_4

$V_0 = C \times PVAF_{0.1,3} = \$1000\ [\{1-(1+0.1)^{-3}\}/0.1] = \$2486.852$

$V_4 = V_0\ \ FVIF_{0.1,4} = V_0\ (1+0.1)^4 = \$2486.852 \times 1.4641 = \$3641.$

D. A More Difficult Example:

1. Question: Suppose I receive $100 at the end of the month and three further $100 payments in three, five and seven months from today. The effective monthly interest rate is 1%. i. What amount could I borrow today using these four $100 payments to repay the loan? ii. What is the APR with monthly compounding? iii. What is the APR with compounding every 2 months?



r=0.01

$1+r_2 = (1+r)^2$ and so $r_2 = 1.01^2 - 1 = 0.0201$.

2. Answer to i.: Since payments are made every 2 months, the appropriate interest rate to use for the present value annuity factor is the effective 2 month rate 2.01%; i.e., use $r_2 = 0.0201$. And since payments are made every 2 months, the present value annuity factor gives the single sum equivalent 2 months before the first payment: here one month before today. So

$V_{-1} = \$100\ PVAF_{0.0201,4} = \$100\ [\{1- 1.0201^{-4}\}/0.0201] = \$100 \times 3.8068 = \$380.68.$

is the amount that could have been borrowed one month earlier. To get the amount that could be borrowed today, use the future value interest factor:

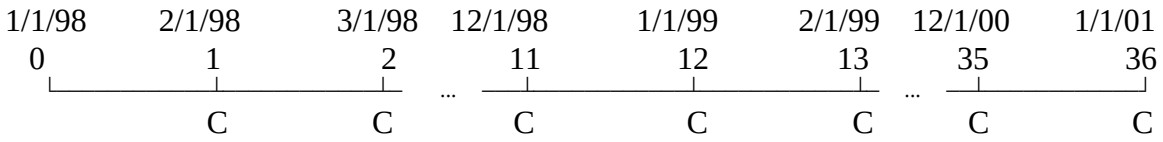
$V_0 = V_{-1}\ FVIF_{0.01,1} = \$380.68\ (1.01) = \$384.49.$

3. Answer to ii. With monthly compounding the APR is 12x1%=12%

4. Answer to iii. With compounding every 2 months, the APR is 6x2.01% = 12.06%.

E. Amortization:

1. Question: Suppose I borrow $5000 on 1st January 1998 at an APR of 18% compounded monthly. i. If I have a three year loan and I make loan repayments at the end of each month, what is my monthly payment? ii. What is the loan balance outstanding after the first loan payment? iii. How much interest accumulates in the first month of the loan?



2. Effective Interest Rate: APR is 18% so the effective monthly rate is (18%/12)=1.5%. Thus, r=0.015.
3. Answer i.

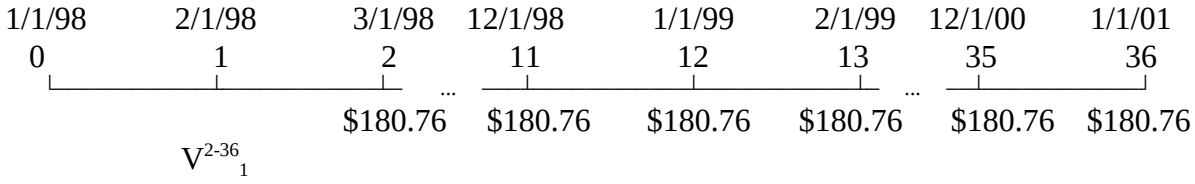
$V_0 = \$5000 = C \times PVAF_{0.015,36} = C\,[\{1-(1+0.015)^{-36}\}/0.015] = C \times 27.660684$; and so

$C = \$5000/27.660684 = \$180.762.$

4. Answer ii.
   a. Loan balance outstanding at time 1 (after the first payment is made):

$\$5000 \times 1.015 - C = \$5075 - \$180.76 = \$4894.24.$

   b. But the loan balance outstanding at time 1 (after the first payment is made) is equal to the single sum equivalent at time 1 of the remaining 35 payments:



$V^{2-36}_1 = \$180.762 \times PVAF_{0.015,35} = \$180.762\,[\{1-(1+0.015)^{-35}\}/0.015] = \$4894.24.$

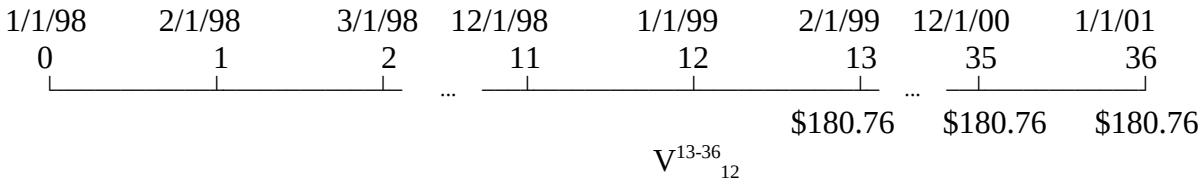
5. Answer iii.
   a. Interest for 1/98 = $5000 x 0.015 = $75.

6. Question (cont): iv. What is the loan balance outstanding after the twelfth loan payment?
7. Answer iv.

a.

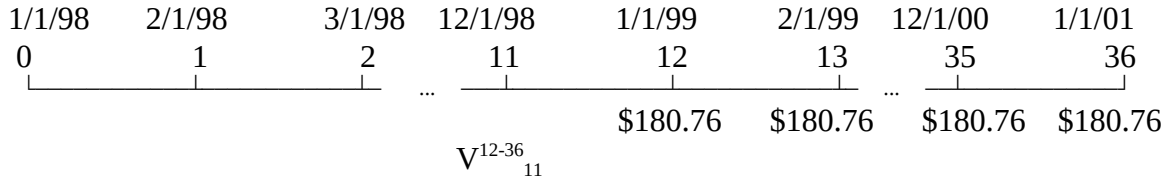
Time	Interest	Balance prior to Payment	Payment	Balance after Payment
0				5000
1	75	5075	180.76198	4894.238
2	73.41357	4967.6516	180.76198	4786.8896
3	71.803344	4858.693	180.76198	4677.931
4	70.168965	4748.0999	180.76198	4567.338
5	68.51007	4635.848	180.76198	4455.0861
6	66.826291	4521.9124	180.76198	4341.1504
7	65.117256	4406.2676	180.76198	4225.5057
8	63.382585	4288.8882	180.76198	4108.1263
9	61.621894	4169.7482	180.76198	3988.9862
10	59.834793	4048.821	180.76198	3868.059
11	58.020885	3926.0799	180.76198	3745.3179
12	56.179768	3801.4977	180.76198	3620.7357

b. Alternatively, the loan balance outstanding after 12 payments is the single sum equivalent at time 12 of the last 24 payments:



$V^{13\text{-}36}_{12} = \$180.762 \times PVAF_{0.015,24} = \$180.762\ [\{1-(1+0.015)^{-24}\}/0.015] = \$3620.736.$

8. Question (cont): v. How much interest accumulates in the twelfth month of the loan?
9. Answer v. To determine how much interest accumulates in the 12th month, need to determine the loan balance outstanding after 11 payments. It is the single sum equivalent at time 11 of the last 25 payments:



$V^{12\text{-}36}_{11}$ = \$180.762 x $PVAF_{0.015,25}$ = \$180.762 [{1-(1+0.015)$^{-25}$}/0.015] = \$3745.32.

So the amount of interest accumulating during the 12th month is \$3745.32 x 0.015 = \$56.180.

X. Particular Cash Flow Pattern: Perpetuity

A. Definition:

1. equal payments made at equal intervals forever.



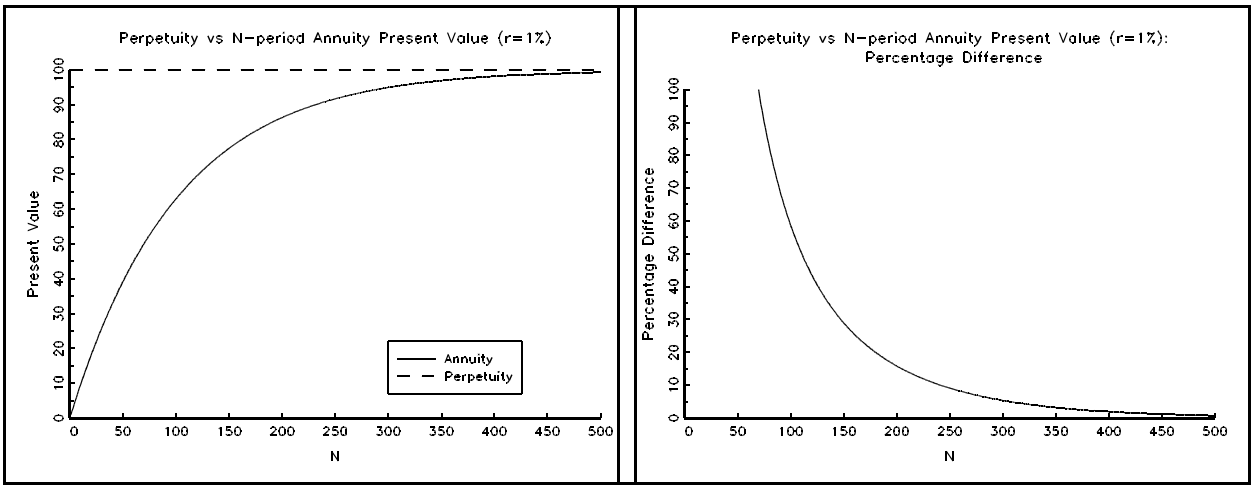
B. Converting to a single sum:

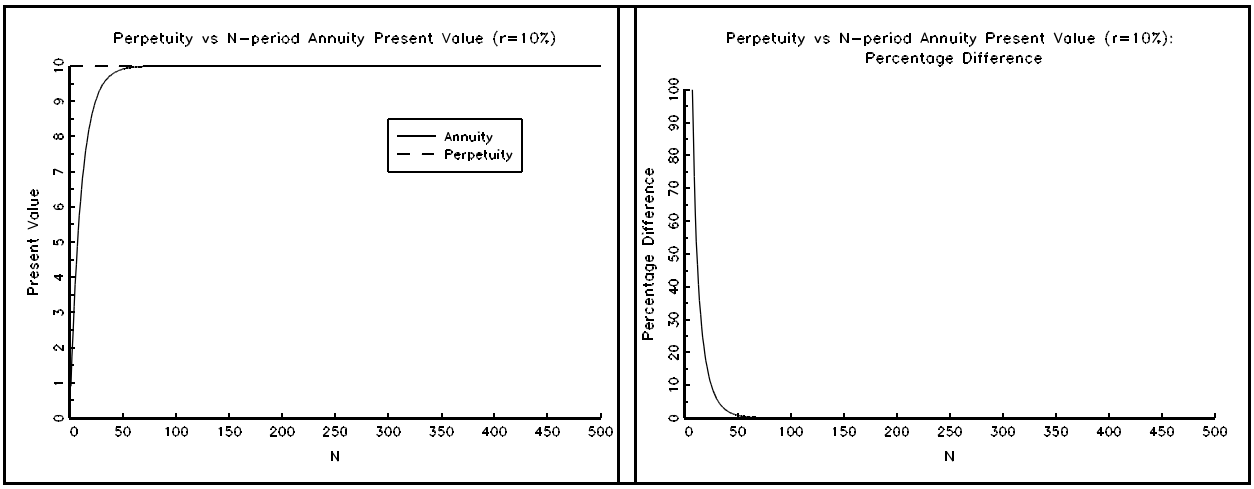
1. can not convert each cash flow to a single sum at a given point in time using the present or future value formulas and then add up these sums to give the value of the annuity at that point in time.
2. Approach:
   a. choose the period so that it is equal to the interval between cash flows.
   b. the perpetuity's first cash flow occurs at (t+1).
   c. use the present value perpetuity factor ($PVPF_r$) which satisfies $V_t = C \times PVPF_r$ and
   d. the effective interest rate over a period is r.
   e. the formula gives the single sum equivalent at the point in time one period before the first cash flow.

(1) $$PVPF_r = \frac{1}{r} .$$

(2) note that

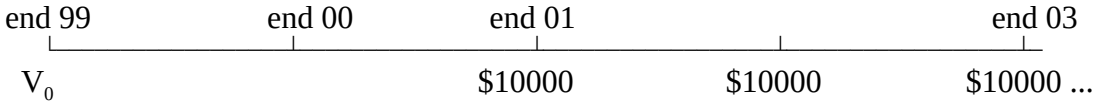
$$PVPF_r = \frac{1}{r} = \lim_{N \to \infty} PVAF_{r,N} = \lim_{N \to \infty} \frac{1 - (1+r)^{-N}}{r} .$$


Perpetuity vs N-period Annuity Present Value (r=1%)
Present Value
N
Annuity
Perpetuity
Perpetuity vs N-period Annuity Present Value (r=1%):
Percentage Difference
Percentage Difference
N


Perpetuity vs N-period Annuity Present Value (r=10%)
Present Value
N
Annuity
Perpetuity
Perpetuity vs N-period Annuity Present Value (r=10%):
Percentage Difference
Percentage Difference
N

C. Example.

1. Question: Mr X wants to set aside an amount of money today that will pay his son and his descendants $10000 at the end of each year forever, with the first payment to be made at the end of 2001. If Mr X can invest at an effective annual rate of 10%, how much would he have to invest today (the 31st December 1999)?



2. Answer:

a. First, calculate how much Mr X needs to have at the end of 2000 using the present value annuity formula; can use this formula because the first payment is at the end of 2000 and the payments are made annually.

$V_1 = \$10000 \times PVPF_{0.1} = \$10000/0.1 = \$100000.$

b. Second, calculate the amount that Mr X must invest at the end of 1999 using the present value formula:

$V_0 = V_1 \times PVIF_{0.1,1} = \$100000/(1+0.1) = \$90909.$

XI. Application

A. U.S. Treasury Notes and Bonds.

1. Introduction.

a. The distinction between notes and bonds is one of original maturity: notes have an original maturity of 1-10 years; bonds have a maturity>10 years.

b. A plain-vanilla bond is characterized by:

(1) Maturity: when the bond will be repaid.

(2) Par or face value: the amount that will be repaid at maturity.

(3) Coupon rate: the rate used in computing the semiannual coupon payments (0.5 x coupon rate x par value gives the semiannual coupon).

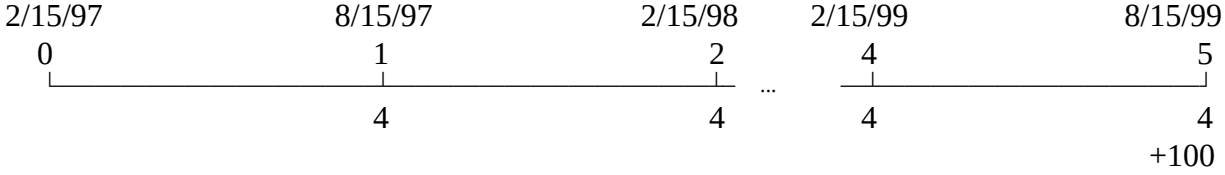
(4) Coupons are either paid on the 15th or at the end of the month.

(5) The quoted prices are on the basis of $100 par, in dollars + 1/32nds.

c. Example: See WSJ clipping for Govt Bonds and Notes on 2/18/97.

Rate	*Maturity Mo/Yr*	*Bid*	*Asked*	*Chg*	*Ask Yld.*
8	Aug 99n	104:28	104:30	-1	5.84

(1) The time line for this bond:



(2) Coupons for this note are paid on the 15th of the month.

(3) The asked price is 104+30/32=104.9375.

2. Yield to maturity (YTM).
   a. Definition.
      (1) YTM is the interest rate such that the present value of the remaining cash flows from the note/bond exactly equals the invoice price.
      (2) The "Ask Yld" in the WSJ is the YTM expressed as an APR with semiannual compounding.
   b. Calculation.
      (1) Suppose the bond has just paid a coupon. Then the YTM expressed as an APR with semi-annual compounding satisfies:

$$V_0 = C \times PVAF_{YTM/2,N} + 100 \times PVIF_{YTM/2,N}$$

where N is the number of coupon payments to maturity and V_0 is the invoice price today.

      (2) If the bond has not just paid a coupon, the calculation is more complicated.
   c. Example (cont): On 2/18/97, Aug 99 note has just paid a coupon. Thus, can use the formula to get the invoice price which will also equal the quoted price:

$$V_0 = 4 \times PVAF_{(5.84/2)\%,5} + 100 \times PVIF_{(5.84/2)\%,5} = 18.361 + 86.597 = 104.958 \approx 104{:}30.$$