Lecture 8: Portfolio Management-2 Risky Assets and a Riskless Asset.

I. Reading.
   A. BKM, Chapter 8: read Sections 8.1 to 8.3.

II. Standard Deviation of Portfolio Return: Two Risky Assets.
   A. Formula:

$$\sigma^2[R_p(t)] = \omega_{1,p}^2\ \sigma[R_1(t)]^2 + \omega_{2,p}^2\ \sigma[R_2(t)]^2 + 2\ \omega_{1,p}\ \omega_{2,p}\ \sigma[R_1(t),\ R_2(t)]$$

where
$\sigma[R_1(t), R_2(t)]$ is the covariance of asset 1's return and asset 2's return in period t;
$\omega_{i,p}$ is the weight of asset i in the portfolio p;
$\sigma^2[R_p(t)]$ is the variance of return on portfolio p in period t.

   B. Example 2 (cont): Consider a portfolio formed at the start of February 1997 with 60% invested in the small firm asset and 40% in Microsoft.
      1. Use data in Lecture 6 p13-15.
      2. What is the portfolio's standard deviation ignoring DP?

$\sigma^2[R_p]$ $= \omega_{Small,p}^2\ \sigma^2[R_{Small}] + \omega_{Msft,p}^2\ \sigma^2[R_{Msft}] + 2\ \omega_{Small,p}\ \omega_{Msft,p}\ \sigma[R_{Small},R_{Msft}]$
$= 0.6^2 \times 3.711^2 + 0.4^2 \times 8.203^2 + 2 \times 0.6 \times 0.4 \times 12.030$
$= 4.958 + 10.766 + 5.774 = 21.498.$

$\sigma[R_p] = \sqrt{\sigma^2[R_p]} = \sqrt{21.498} = 4.637.$

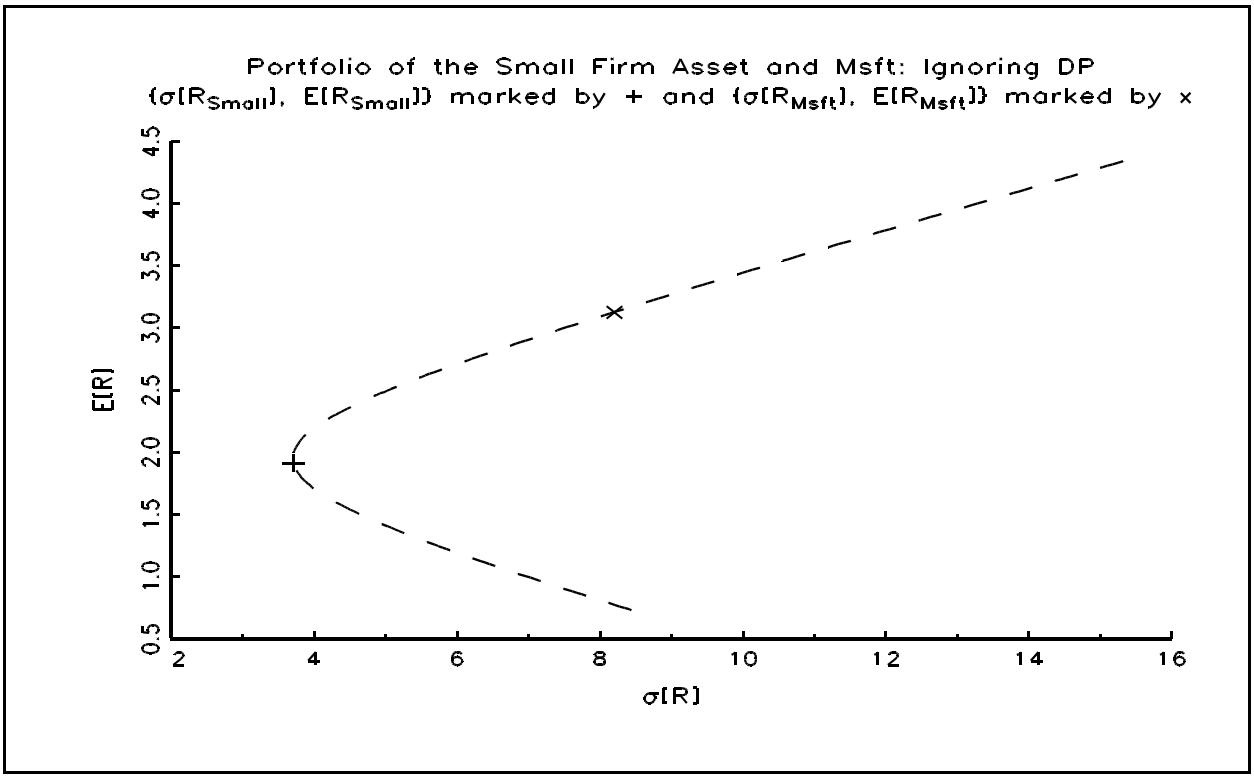
      3. Obtain expected portfolio return using the formula in Lecture 7.

$E[R_p] = \omega_{Small,p}\ E[R_{Small}] + \omega_{Msft,p}\ E[R_{Msft}]$
$= 0.6 \times 1.912 + 0.4 \times 3.126$
$= 2.398$

III. Graphical Depiction: Two Risky Assets.

A. The standard deviation of return on a portfolio consisting of the small firm asset and Microsoft and its expected return can be indexed by the weight of the small firm asset in the portfolio. The curve is known as the portfolio possibility curve.

$\omega_{Small,p}$	$\omega_{Msft,p}$	$\sigma[R_p(t)]$	$E[R_p(t)]$
-0.2	1.2	9.574%	3.369%
0.0	1.0	8.203%	3.126%
0.2	0.8	6.889%	2.883%
0.4	0.6	5.675%	2.641%
0.6	0.4	4.637%	2.398%
0.8	0.2	3.919%	2.155%
1.0	0.0	3.711%	1.912%
1.2	-0.2	4.093%	1.670%



IV. Impact of Correlation: Two Risky Asset Case

A. Standard Deviation Formula.

1. Can be rewritten in terms of correlation rather than covariance (using the definition of correlation):

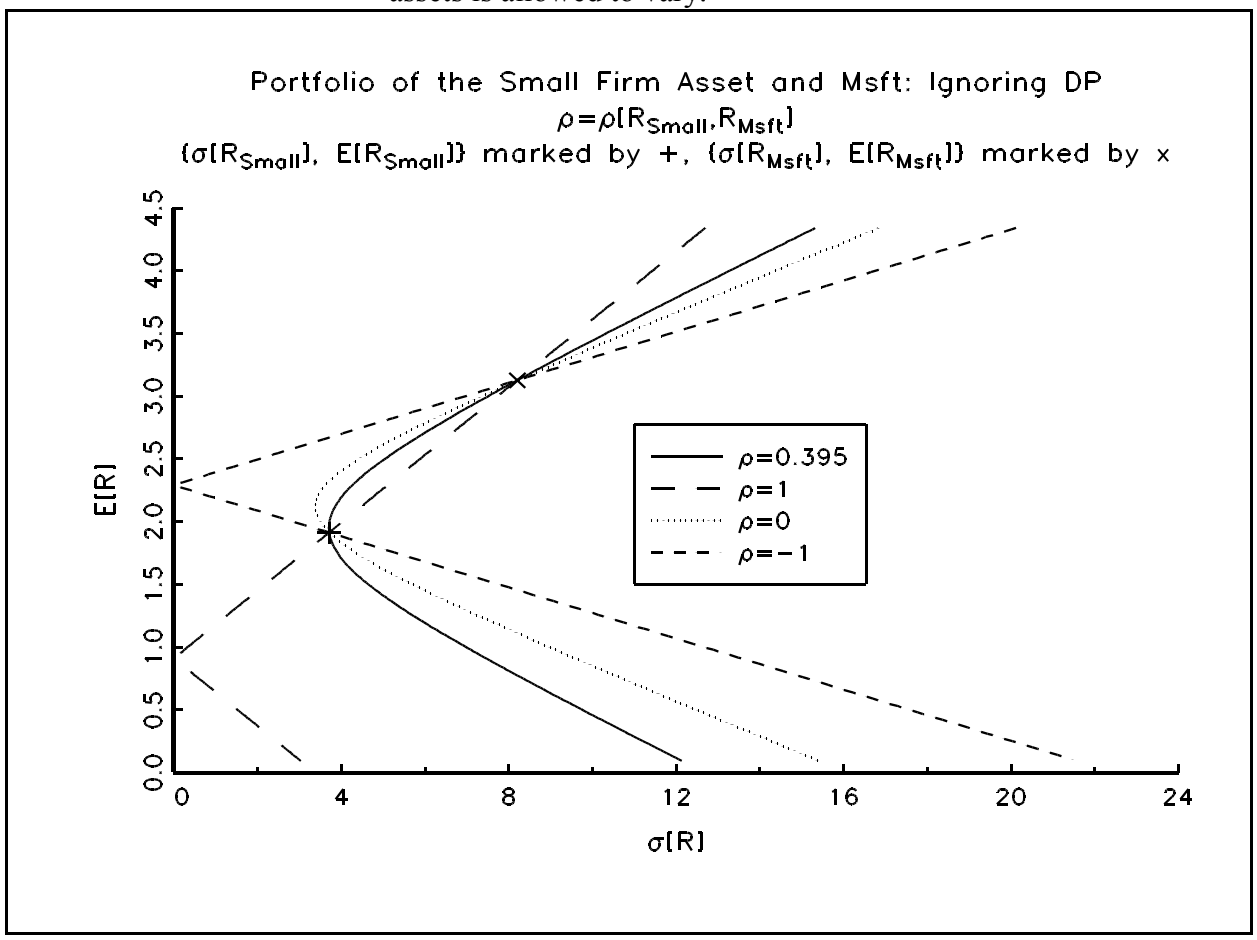
$$\sigma^2[R_p(t)] = \omega_{1,p}^2\ \sigma[R_1(t)]^2 + \omega_{2,p}^2\ \sigma[R_2(t)]^2 + 2\ \omega_{1,p}\ \omega_{2,p}\ \rho[R_1(t),\ R_2(t)]\ \sigma[R_1(t)]\ \sigma[R_2(t)]$$

where $\rho[R_1(t), R_2(t)]$ is the correlation of asset 1's return and asset 2's return in period t;

2. For a given portfolio with $\omega_{1,p} > 0$ and $\omega_{2,p} > 0$ and $\sigma[R_1(t)]$ and $\sigma[R_1(t)]$ fixed, $\sigma[R_p(t)]$ decreases as $\rho[R_1(t), R_2(t)]$decreases.

B. Example 2 (cont):

a. Suppose the E[R] and σ[R] for the small firm asset and for Microsoft remain the same but the correlation between the two assets is allowed to vary:



V. Portfolio Choice: the Two Risky Asset Portfolio.

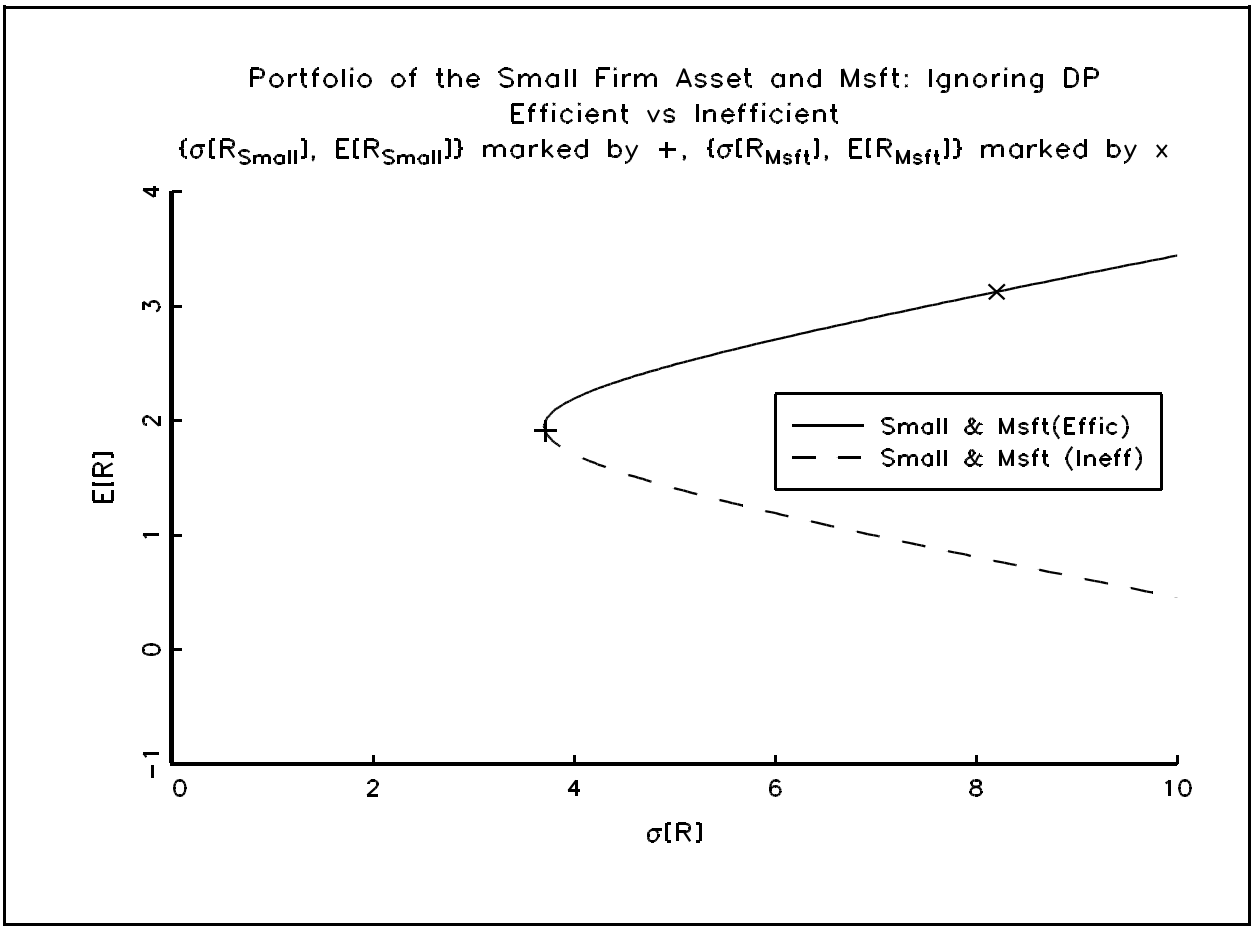
A. A risk averse investor is not going to hold any combination of the two risky assets on the negative sloped portion of the portfolio possibility curve.

1. So the negative-sloped portion is known as the inefficient region of the curve.
2. And the positive-sloped portion is known as the efficient region of the curve.

B. The exact position on the efficient region that an individual holds depends on her tastes and preferences.

C. Example 2 (cont): The portfolio possibility curve for the small firm asset and Microsoft can be divided into its efficient and inefficient regions.

1. Any risk averse individual combining the small firm asset with Microsoft wants to lie in the efficient region: so wants to invest a positive fraction of her portfolio in Microsoft.



VI. Portfolio Choice: Combining the Two Risky Asset Portfolio with the Riskless Asset.

A. Two-step Decision Process:

1. What is the preferred weights of the two risky assets in the risky portfolio?
   a. all risk averse individuals want access to the CAL with the largest slope; this involves combining the riskless asset with the same risky portfolio (✚ in the graph below).
   b. this same risky portfolio is the one whose CAL is tangential to the portfolio opportunity curve; this is why ✚ is known as the tangency portfolio (denoted T).
   c. Can calculate the weight of risky asset 1 in the tangency portfolio T using the following formula:

$$\omega_{1,T} = \frac{\sigma[R_2]^2\ E[r_1] - \sigma[R_1,\ R_2]\ E[r_2]}{\left[\begin{array}{l}\{\sigma[R_2]^2\ E[r_1] - \sigma[R_1,\ R_2]\ E[r_2]\} \\ + \{\sigma[R_1]^2\ E[r_2] - \sigma[R_1,R_2]\ E[r_1]\}\end{array}\right]}$$

where $r_i = R_i - R_f$ is the excess return on asset i (in excess of the riskless rate).

2. What is the preferred weights of the risky portfolio T and the riskless asset in the individual's portfolio?
   a. the weight of T (✚) in an individual's portfolio $\omega_{T,p}$ depends on the individual's tastes and preferences.

B. Example 2 (cont): Suppose I form a portfolio (ignoring DP) of the small firm asset, Microsoft and T-bills.

1. What is the preferred weights of the two risky assets in the risky portfolio?
   a. Graph suggests that the risky asset portfolio I want to hold has positive weights invested in the small firm asset and in Microsoft (since ✚ is between + and **x** on the portfolio possibility curve for the small firm asset and Microsoft).
   b. Can calculate the weight of the small firm asset in the tangency portfolio using the following formula:

$$\omega_{Small,T} = \frac{\sigma[R_{Msft}]^2\ E[r_{Small}] - \sigma[R_{Small}, R_{Msft}]\ E[r_{Msft}]}{\left[\begin{array}{l}\{\sigma[R_{Msft}]^2\ E[r_{Small}] - \sigma[R_{Small}, R_{Msft}]\ E[r_{Msft}]\} \\ + \{\sigma[R_{Small}]^2\ E[r_{Msft}] - \sigma[R_{Small}, R_{Msft}]\ E[r_{Small}]\}\end{array}\right]}$$

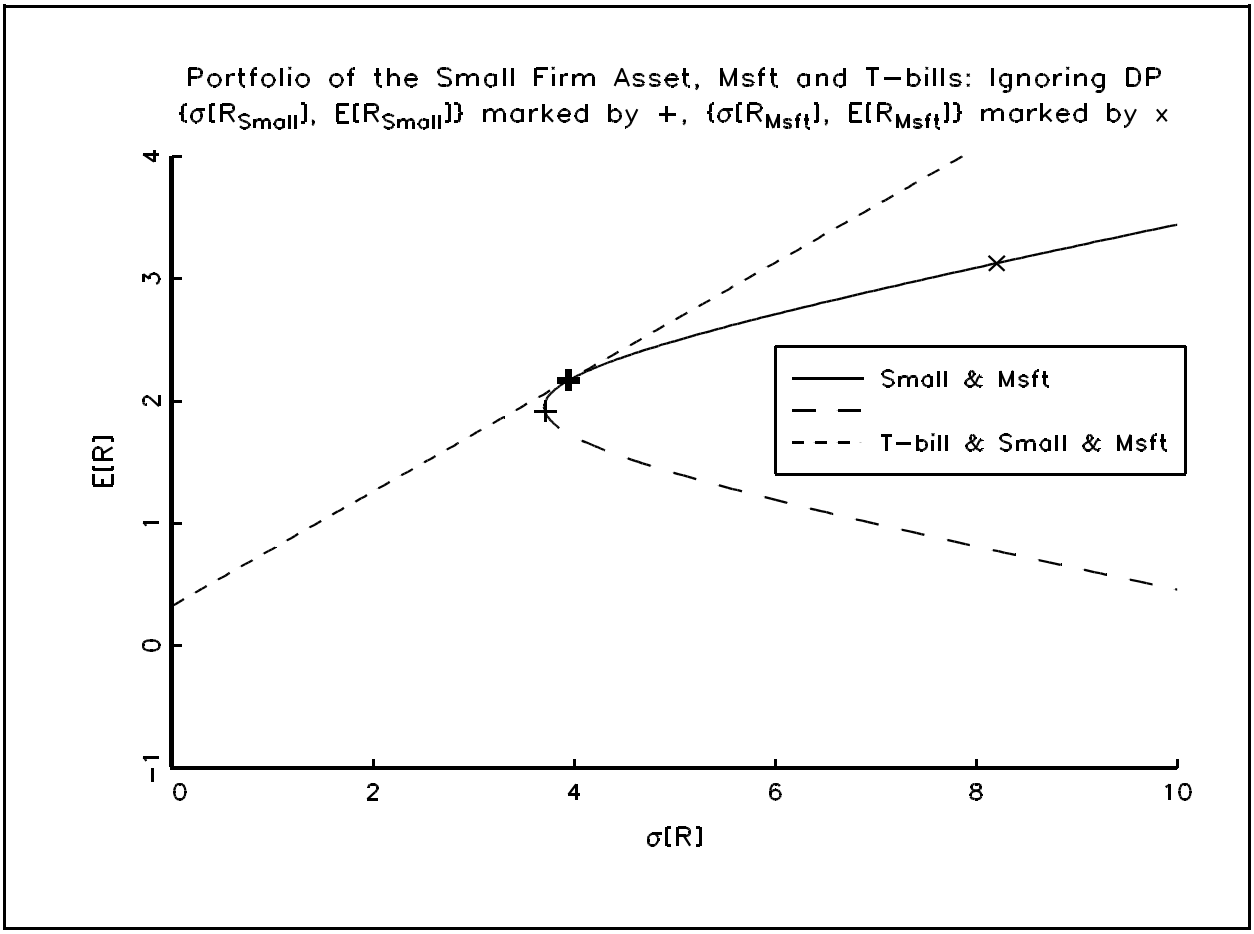
   c. Now (using Lecture 6 p13-15)

$E[r_{Small}]$ = 1.912 - 0.323 = 1.589.
$E[r_{Msft}]$ = 3.126 - 0.323 = 2.803.
$\sigma[R_{Small}]^2$ = 3.711 x 3.711 = 13.772.
$\sigma[R_{Msft}]^2$ = 8.203 x 8.203 = 67.289.
$\sigma[R_{Msft}, R_{Small}]$ = 12.030.

$$\omega_{Small,T} = \frac{67.289 \times 1.589 - 12.030 \times 2.803}{\left[\begin{array}{l}\{67.289 \times 1.589 - 12.030 \times 2.803\} \\ + \{13.772 \times 2.803 - 12.030 \times 1.589\}\end{array}\right]}$$

= 73.202 / [73.202 + 19.487] = 0.790.

2. What is the preferred weights of the risky portfolio T and the riskless asset in the individual's portfolio?
   a. Depends on the tastes and preferences of the particular individual.



3. Suppose Individual Y wants to invest 75% in the tangency portfolio (✚) and 25% in T-bills. What is the weight of the small firm asset and of Microsoft in Y's total portfolio?

   a. Use the following formula:

$\omega_{i,p} = \omega_{i,T}\,\omega_{T,p}$

where

$\omega_{i,T}$ is the weight of risky asset i in the tangency portfolioT.
$\omega_{i,p}$ is the weight of risky asset i in the total portfolio p.
$\omega_{T,p}$ is the weight of portfolio T in the total portfolio p.

   b. So, the answer is:

$\omega_{Small,p} = \omega_{Small,T}\,\omega_{T,p} = 0.79 \times 0.75 = 0.5925.$
$\omega_{Msft,p} = \omega_{Msft,T}\,\omega_{T,p} = 0.21 \times 0.75 = 0.1575.$

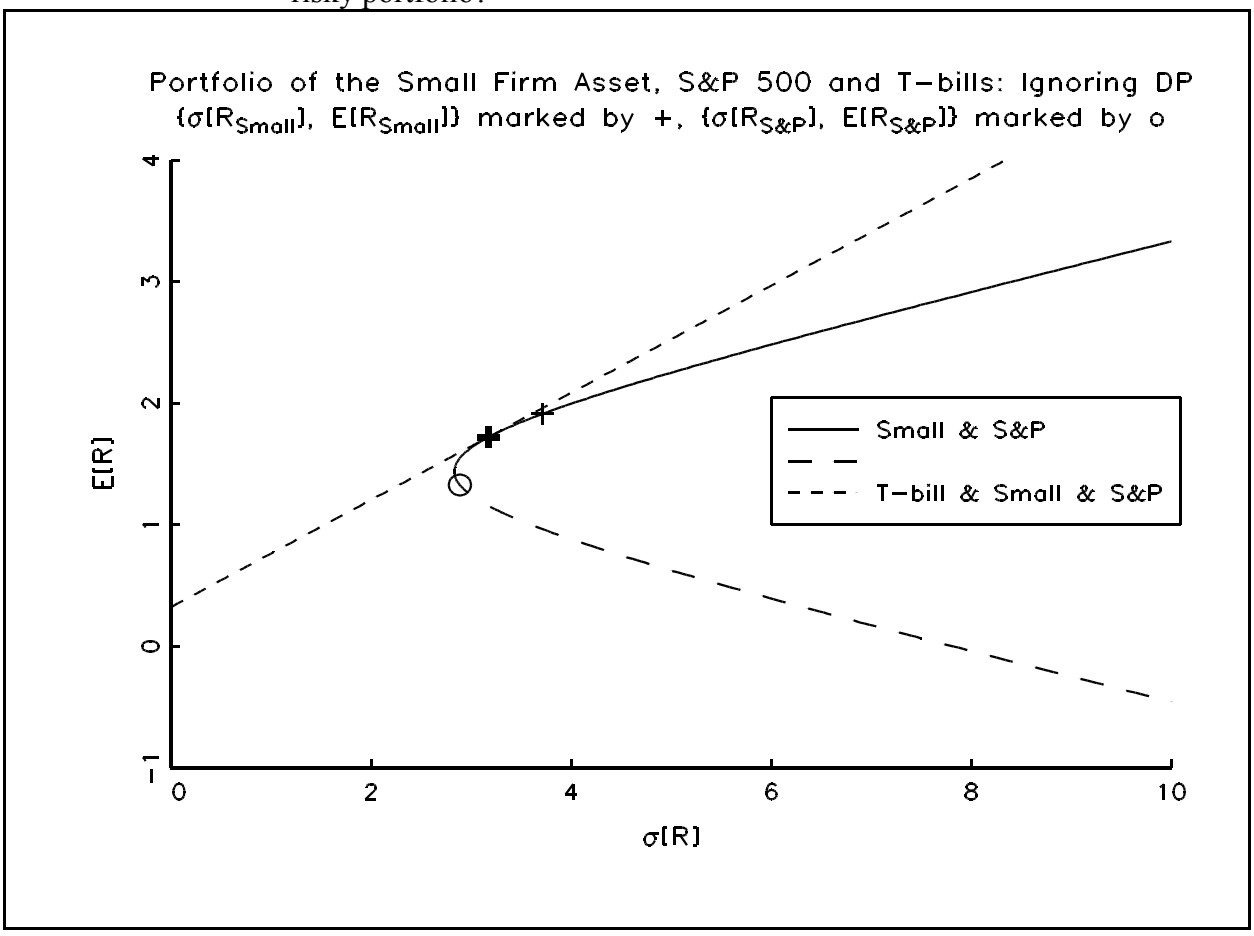
VII. Applications.

A. Adding a New Stock: The two-risky-asset formulas can be used to assess the impact of adding a new stock to a portfolio or varying the weight of an existing stock in the portfolio.

1. Example 2 (cont): Above considered the impact of adding Microsoft to the small firm fund (ignoring DP).

B. Asset Allocation between Two Broad Classes of Assets: The two-risky-asset formulas can be used to determine how much to invest in each of two broad asset classes.

1. Example 2 (cont): If I intend to form a risky portfolio from the small firm asset and the S&P 500 (ignoring DP) and then combine that risky portfolio with the riskless asset, what weight will the small firm asset have in the risky portfolio?



2. Can see from the graph that using historical data from 1/91 to 12/95 to approximate the return distribution for 2/97, would lead a risk averse individual to hold a risky portfolio with positive weights in both the small firm asset and in the S&P 500 ($\omega_{Small,T} = 0.6687$ using formula above).
3. Weight of the small firm asset in the tangency portfolio $\omega_{Small,T}$ is sensitive to the $E[R_{Small}]$:
   a. As $E[R_{Small}]$ declines so does $\omega_{Small,T}$ holding $E[R_{S\&P}]$, the standard deviations and correlation fixed:

$E[R_{Small}]$	$E[R_{S\&P}]$	$\omega_{Small,T}$
1.912%	1.326%	66.87%
1.619%	1.326%	44.24%
1.326%	1.326%	19.09%

   b. This sensitivity of $\omega_{Small,T}$ to changes in $E[R_{Small}]$ explains why the small firm effect is of interest to practitioners.

C. International Diversification: The two-risky-asset formulas can also be used when deciding how much to invest in an international equity fund and how much in a U.S. based fund.