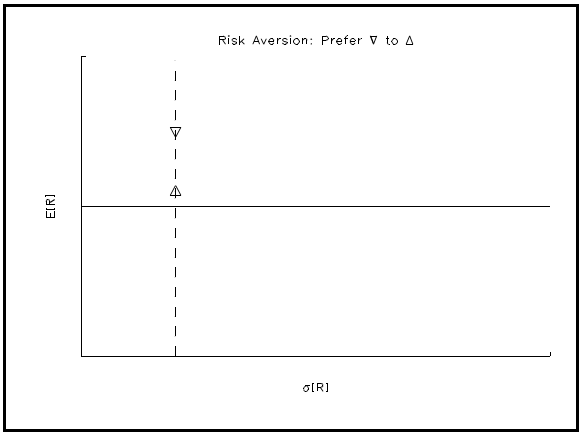
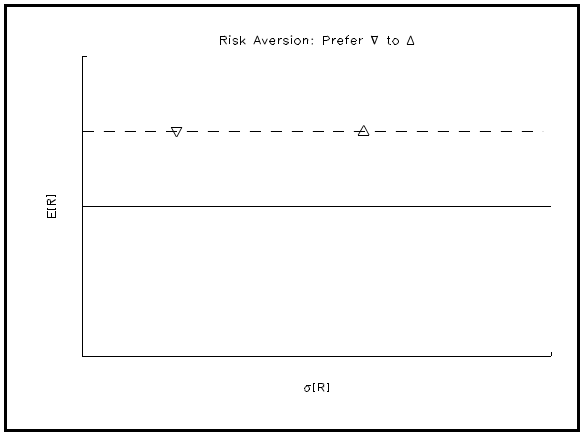
Lecture 7: Portfolio Management-A Risky and a Riskless Asset.

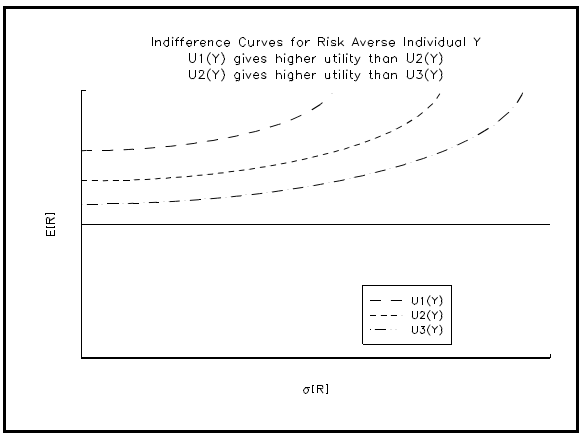
I. Reading.
- A. BKM, Chapter 6: read this chapter (though Section 6.1 is more detailed than is needed); ignore the Appendices.
- B. BKM, Chapter 7: skim Sections 7.1 and 7.2; read Section 7.3; read lightly Sections 7.4 and 7.5.

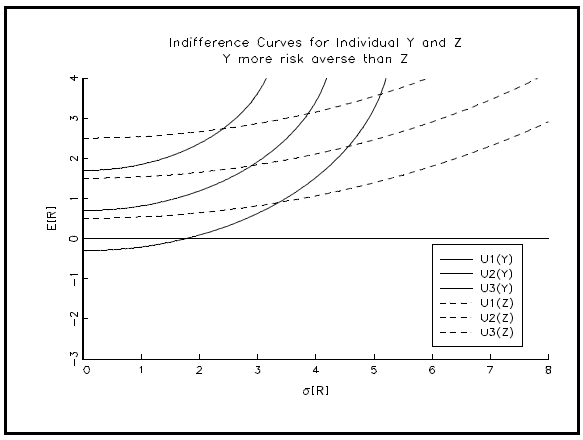
II. Investor Preferences.
- A. Summarizing Tastes and Preferences.
  1. For the moment, assume a one period setting.
  2. For certain return distributions (e.g., multivariate normal), individual preferences can be completely characterized by:
     a. Expected Return over the Period, E[R].
     b. Standard Deviation of Return over the Period, σ[R].
  3. In other words, individuals only care about their expected portfolio return and about their portfolio's standard deviation.
- B. Risk Aversion.
  1. One of the cornerstones of modern finance is that individuals are risk averse (and prefer more to less).
  2. For any risk averse individual, the following is true:
     a. For a given expected portfolio return, prefer a portfolio with a lower standard deviation of return.
     b. For a given standard deviation of portfolio return, prefer a portfolio with a higher expected return.





3. Use indifference curves to represent an individual's tastes and preferences:





a.

At all points on an indifference curve, the investor enjoys the same level of utility.

b. In {Standard Deviation of Return, Expected Return} space, a risk averse individual's indifference curves have positive slopes: Since a risk averse individual likes mean but dislikes standard deviation, the only way the individual can accept more standard deviation and maintain the same level of utility is if she is given a higher expected return.

c. For any individual, as you move north in {σ[R], E[R]} space, utility is increasing.

d. For any individual, her indifference curves can not cross since that would imply that a particular {σ[R], E[R]} combination was associated with two levels of utility.

4. However, the trade-off between risk and return for any two risk averse individuals may be completely different (see individuals Y and Z above).

a. Individual Y is more risk averse than Z since at any point in {σ[R], E[R]} space,Y's indifference curve has a steeper slope.

III. Expected Portfolio Return: General Formula

A. Formula: holds for any number of assets and with or without the risky asset as one of the assets:

$$E[R_p(t)] = \omega_{1,p} E[R_1(t)] + \omega_{2,p} E[R_2(t)] + ... + \omega_{N,p} E[R_N(t)]$$

where

N is the number of assets in the portfolio;
$E[R_i(t)]$ is the expected return on asset i in period t;
$\omega_{i,p}$ is the weight of asset i in the portfolio p at the start of period t;
$E[R_p(t)]$ is the expected return on portfolio p in period t.

B. Example 1 (cont): Consider a portfolio with 80% invested in Ford and the remaining 20% invested in T-bills.

$E[R_p] = \omega_{Ford,p} E[R_{Ford}] + \omega_{T\text{-}bill,p} E[R_{T\text{-}bill}]$
$= 0.8 \times 9.6\% + 0.2 \times 5\% = 8.68\%.$

C. Example 2 (cont): Consider a portfolio formed at the end of January 1997 with 60% invested in the small firm portfolio and the remaining 40% invested in 1 month T-bills.

1. What is the portfolio's expected return ignoring DP(start Feb)?

$E[R_p] = \omega_{Small,p} E[R_{Small}] + \omega_{T\text{-}bill,p} E[R_{T\text{-}bill}]$
$= 0.6 \times 1.912\% + 0.4 \times 0.323\% = 1.2764\%.$

2. What is the portfolio's expected return using DP(start Feb)?

$E[R_p] = \omega_{Small,p} E[R_{Small}] + \omega_{T\text{-}bill,p} E[R_{T\text{-}bill}]$
$= 0.6 \times -1.509\% + 0.4 \times 0.323\% = -0.7762\%.$

3. Using the starting DP to help determine expected return can make a big difference.

IV. Standard Deviation of Portfolio Return: One Risky Asset and a Riskless Asset.

A. Formula: holds when one asset is risky and the other is riskless:

$$\sigma[R_p(t)] = |\omega_{i,p}| \, \sigma[R_i(t)]$$

where

$\sigma[R_i(t)]$ is the standard deviation of return on risky asset i in period t;
$|\omega_{i,p}|$ is the absolute value of the weight of asset i in the portfolio p;
$\sigma[R_p(t)]$ is the standard deviation of return on portfolio p in period t.

B. Example 1(cont): Consider the portfolio with 80% invested in Ford and the remaining 20% invested in T-bills.

$\sigma[R_p] = |\omega_{Ford,p}| \, \sigma[R_{Ford}]$
$= 0.8 \times 15.5897\% = 12.4718\%.$

C. Example 2 (cont): Consider the portfolio formed at the end of January 1997 with 60% invested in the small firm portfolio and the remaining 40% invested in 1 month T-bills.

1. What is the portfolio's standard deviation ignoring DP(start Feb)?

$\sigma[R_p] = |\omega_{Small,p}| \, \sigma[R_{Small}]$
$= 0.6 \times 3.711\% = 2.2266\%.$

2. What is the portfolio's standard deviation using DP(start Feb)?

$\sigma[R_p] = |\omega_{Small,p}| \, \sigma[R_{Small}]$
$= 0.6 \times 3.549\% = 2.1294\%.$

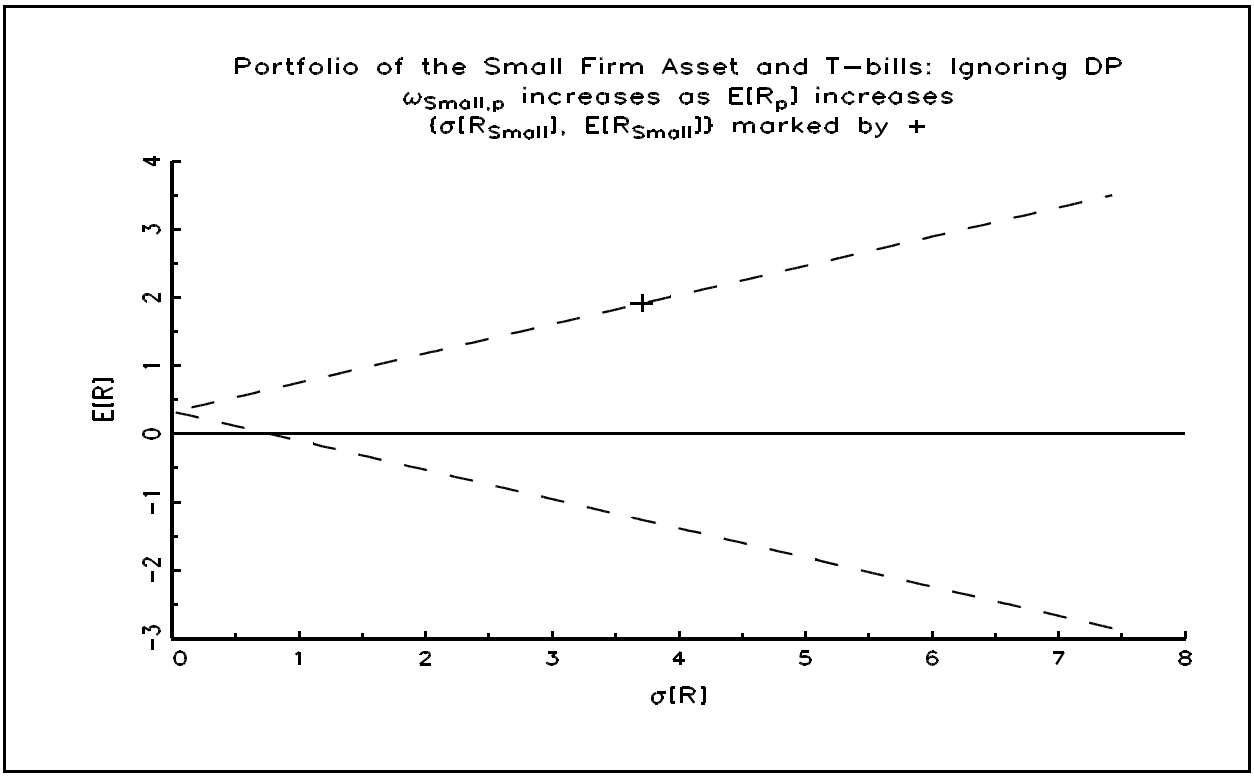
3. Portfolio standard deviation is largely unaffected by using the starting DP to predict return.
4. Note that using these formulas are just as easy for the real data as for the mock data.

V. Graphical Depiction: Portfolio Expected Return and Standard Deviation.

A. Example 2 (cont): The standard deviation of return on a portfolio consisting of the small firm asset and T-bills and its expected return can be indexed by the weight of the small firm asset in the portfolio:

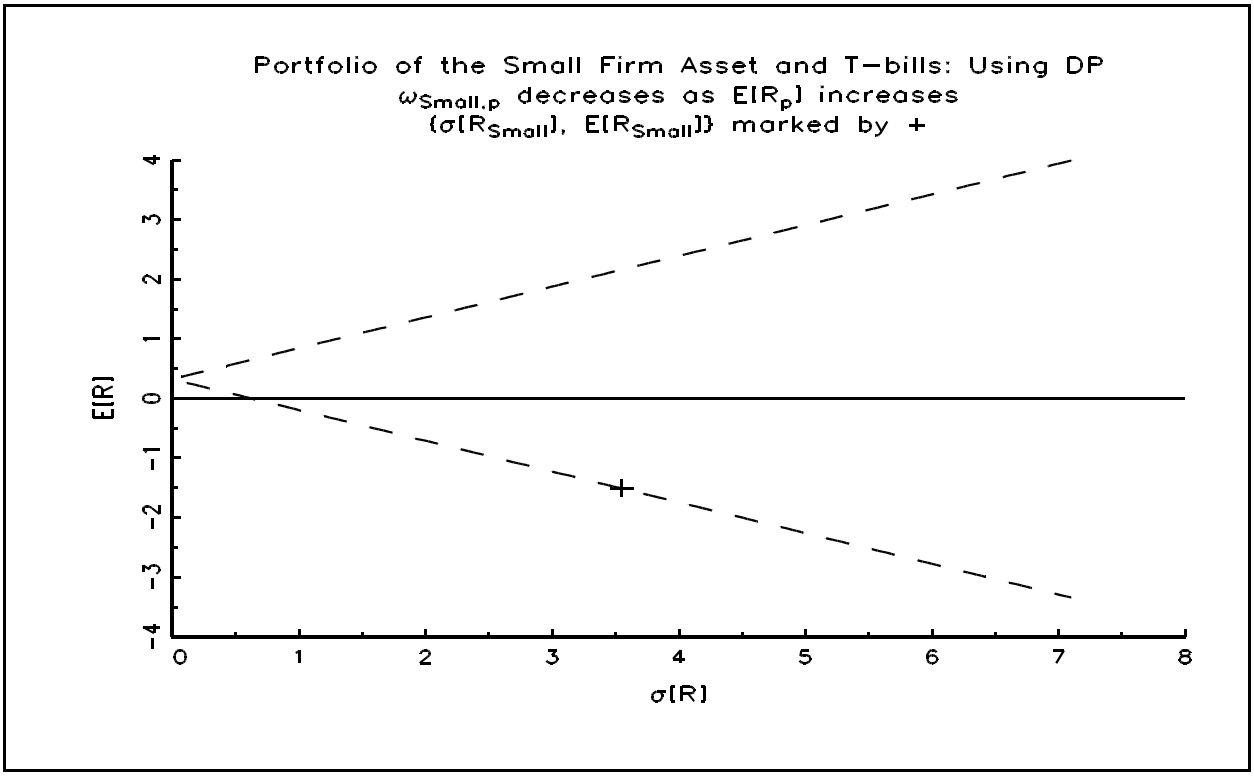
1. Ignoring DP(start Feb):

$\omega_{Small,p}$	$\omega_{T\text{-}bill,p}$	$\sigma[R_p(t)]$	$E[R_p(t)]$
-0.2	1.2	0.742%	0.005%
0.0	1.0	0.000%	0.323%
0.2	0.8	0.742%	0.641%
0.4	0.6	1.484%	0.959%
0.6	0.4	2.227%	1.277%
0.8	0.2	2.969%	1.595%
1.0	0.0	3.711%	1.912%
1.2	-0.2	4.453%	2.230%



2. Using DP(start Feb):

$\omega_{Small,p}$	$\omega_{T\text{-}bill,p}$	$\sigma[R_p(t)]$	$E[R_p(t)]$
-0.2	1.2	0.710	0.689
0.0	1.0	0.000	0.323
0.2	0.8	0.710	-0.043
0.4	0.6	1.420	-0.410
0.6	0.4	2.129	-0.776
0.8	0.2	2.839	-1.143
1.0	0.0	3.549	-1.509
1.2	-0.2	4.259	-1.875

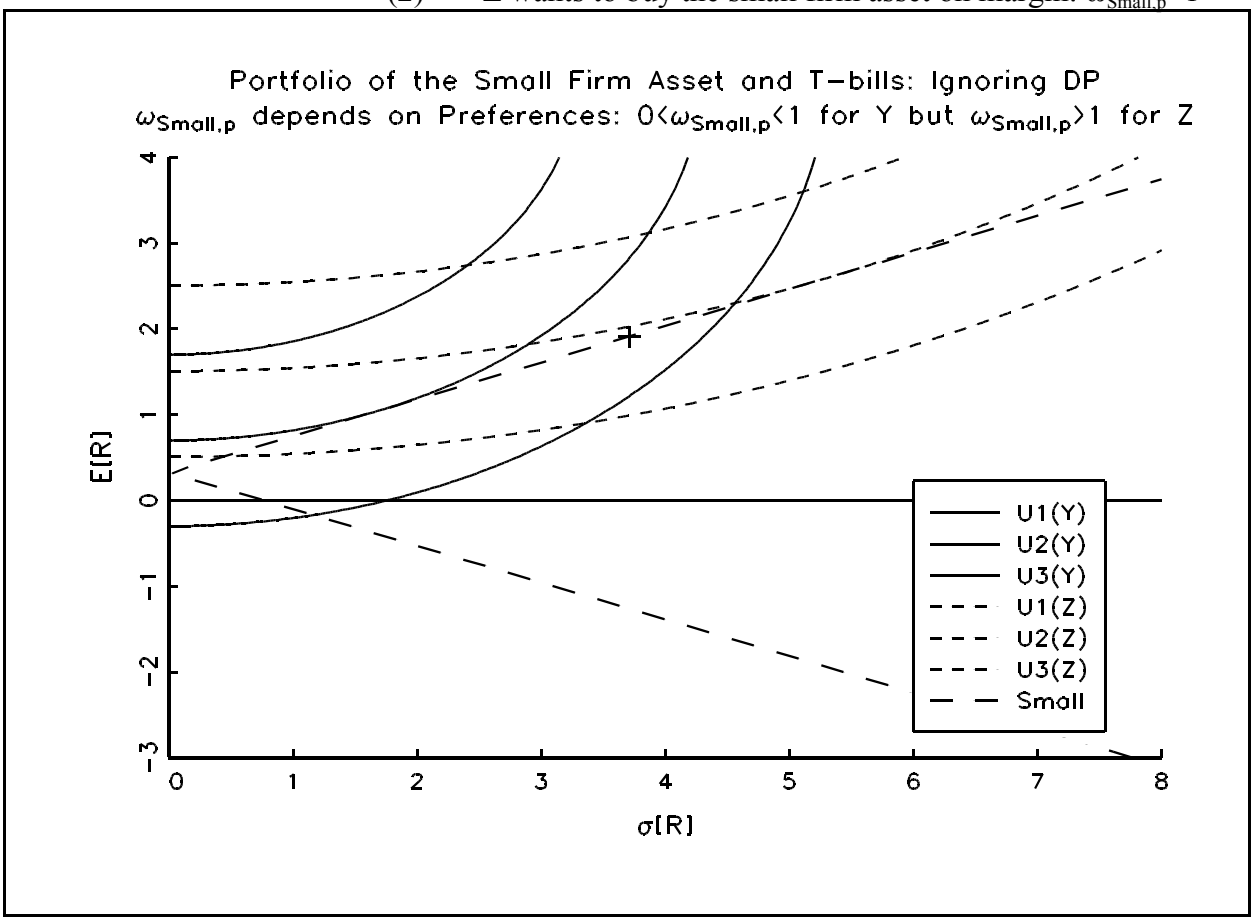


VI. Portfolio Management: One Risky Asset and a Riskless Asset.

A. How would a risk averse investor choose the portfolio weights for a portfolio consisting solely of the riskless asset and a given risky asset?

1. For any point on the negative sloped part of the curve, a risk averse individual is going to prefer at least one point on the positive sloped part of the curve (the one with the same standard deviation and a higher expected return).
2. So if the expected return on a risky asset exceeds the riskless rate, an individual forming a portfolio using only that asset and the riskless asset will not want to short sell the risky asset (not want $\omega_{Risky,p}<0$); but the individual may want to buy it on margin (may want $\omega_{Risky,p}>1$).
    a. Example 2 (cont): Combining the small firm portfolio with T-bills ignoring DP.
3. So if the expected return on a risky asset is less than the riskless rate, an individual forming a portfolio using only that asset and the riskless asset will want to short sell the risky asset (will want $\omega_{Risky,p}<0$).
    a. Example 2 (cont): Combining the small firm portfolio with T-bills using DP.

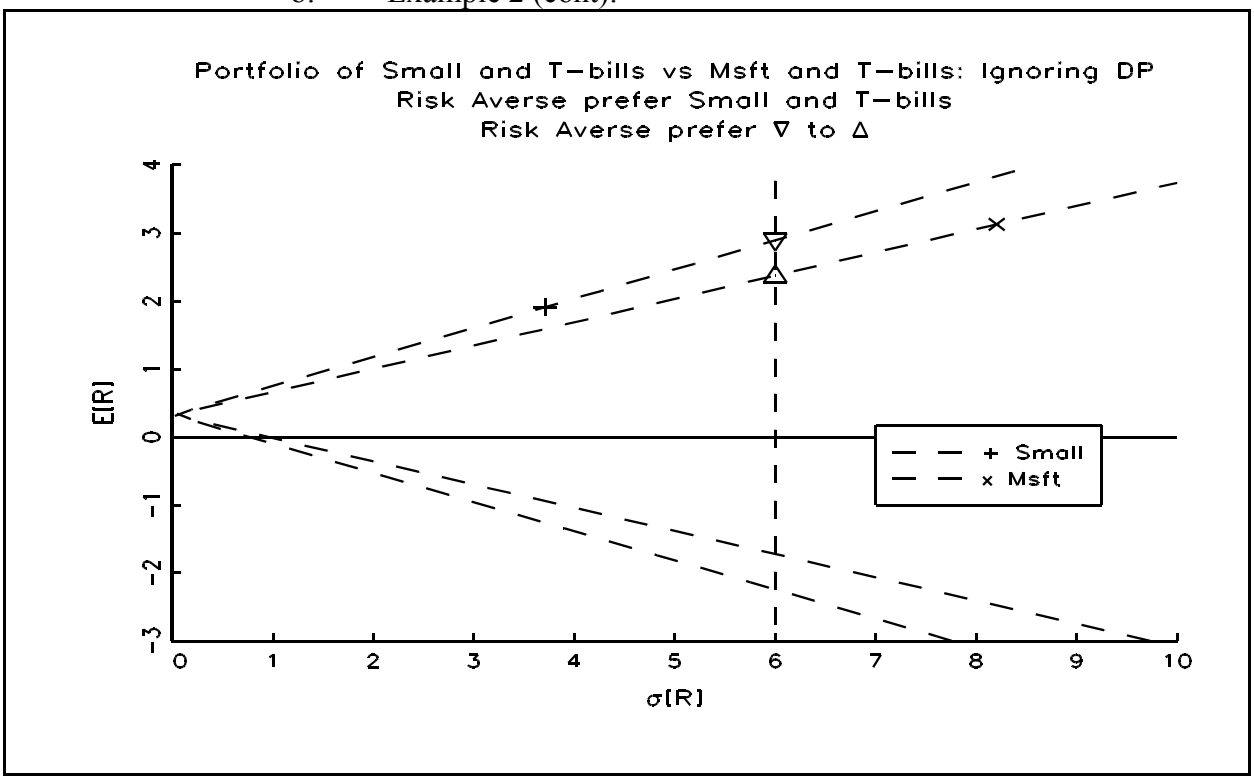
4. The exact weight that the individual wants to hold of the risky asset depends on her attitudes to risk; different individuals will choose to hold different amounts of the risky asset.
   a. Example 2 (cont): Combining the small firm asset with T-bills ignoring DP.
      (1) Y wants to hold positive amounts of both the small firm asset and T-bills: $0<\omega_{Small,p}<1$.
      (2) Z wants to buy the small firm asset on margin: $\omega_{Small,p}>1$



5. The positive sloped line is called the Capital Allocation Line (CAL).

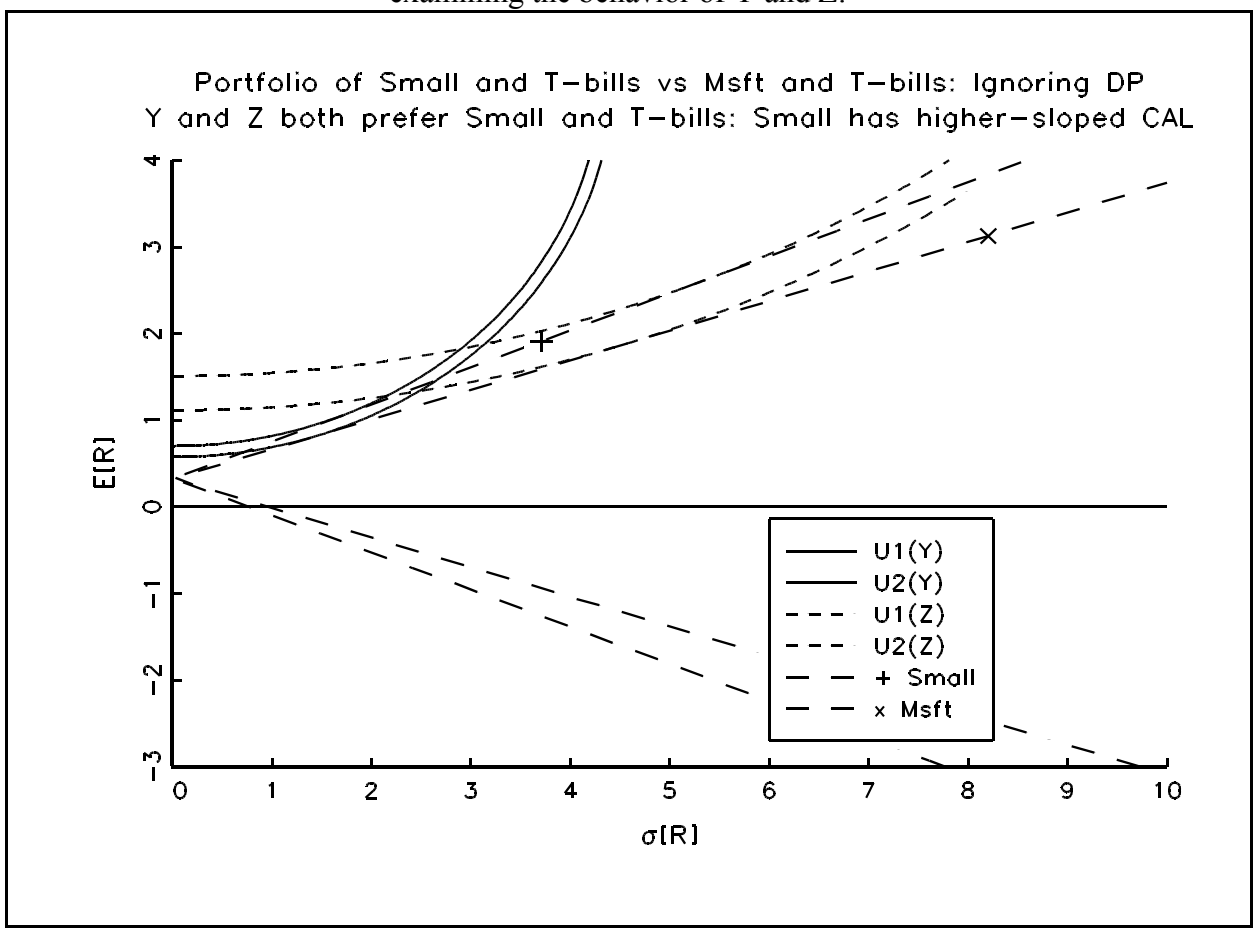
B. If a risk averse investor could use either risky asset A or risky asset B in combination with the riskless asset, how would the investor decide whether to use risky asset A or to use risky asset B?

1. Example 2 (cont): If a risk averse investor could use either the small firm asset or Microsoft in combination with the riskless asset to form a portfolio, how would the investor decide whether to use the small firm asset or to use Microsoft (ignoring DP)?
2. Irrespective of risk preferences, the individual prefers the risky asset whose CAL has the highest slope.
   a. For any point on the lower sloped line, a risk averse investor prefers at least one point on the higher sloped line (the point with the same standard deviation but a higher expected return).
   b. Example 2 (cont):



3. In general, the slope of any risky asset i's CAL is given by slope[CAL_i] = |E[R_i] - R_f| / σ[R_i].

4. Example 2 (cont): Calculate the slope of the CAL for the small firm asset and Microsoft ignoring DP:
   a. slope[CAL_{Small}] = {1.912-0.323}/3.711 = 0.428;
      slope[CAL_{Msft}] = {3.126-0.323}/8.203 = 0.342;
      slope[CAL_{Small}] > slope[CAL_{Msft}].
   b. A risk averse individual's preference for using the risky asset with the highest-sloped CAL (the small firm asset) can also be seen by examining the behavior of Y and Z:



   c. Both individual Y and individual Z can attain higher utility holding the small firm asset rather than Microsoft in combination with the riskless asset.