Lecture 6: Portfolio Management-Characterizing the Return Distribution

I. Reading
- A. BKM Appendix: A1: pgs 893-898, A2, A3.
- B. BKM Chapter 5: Section 5.2.

II. Examples
- A. Example 1: "Mock" Data

State	Probability (pr)	Ford	GM	Reebok	T-Bill
BOOM (BM)	50%	25%	22%	9%	5%
RECOVERY (RC)	30%	-3%	2%	9%	5%
DEPRESSION (DP)	20%	-10%	-19%	3%	5%

- B. Example 2: Real Data
  1. Suppose an investor is interested in forming a portfolio at the end of January 1997 from the following assets:
     a. IBM equity.
     b. Apple equity.
     c. Microsoft equity.
     d. Nike equity.
     e. ADM equity.
     f. S&P 500 index.
     g. small firm portfolio.
     h. long-term government bond portfolio.
     i. U.S. one month t-bill.
  2. The investor also knows the dividend yield on the S&P 500 index at the end of January (DP).
  3. The investor could use historical return data to approximate the joint probability distribution:
     a. each month's realization could be treated as a state.
     b. each state is equally likely; states that occur more frequently will occur more frequently in the data.
     c. use historical data from 1/91 to 12/95; so each realization has a 1/60 probability of occurring.
     d. here, abstract from the issue that this historical distribution is just an approximation of the true return distribution; instead, we will assume that this historical distribution is the true distribution.

III. How to calculate expected return.

A. Formula: If there are K possible states: s1,s2,...,,sK,

$E[R_{Asset}]$ = Expected Return on the Asset
= pr(s1) x R_{Asset}(s1) +pr(s2) x R_{Asset} (s2) + ... + pr(sK) x R_{Asset}(sK)

where R_{Asset} (s) is the return on the Asset in state s; and
pr(s) is the probability of state s.

B. Example 1:

1. Formula: There are 3 states. So,

$E[R_{Asset}]$ = Expected Return on the Asset
= pr(BM) x R_{Asset}(BM) +pr(RC) x R_{Asset}(RC) + pr(DP) x R_{Asset}(DP)

2. Calculations:

$E[R_{Ford}]$ = 0.50 x 25% + 0.30 x -3% + 0.20 x -10% = 9.6%.
$E[R_{GM}]$ = 0.50 x 22% + 0.30 x 2% + 0.20 x -19% = 7.8%.
$E[R_{Reebok}]$ = 0.50 x 9% + 0.30 x 9% + 0.20 x 3% = 7.8%.
$E[R_{T\text{-}bill}]$ = 0.50 x 5% + 0.30 x 5% + 0.20 x 5% = 5%.

C. Example 2: There are 60 states.

Asset	E [R]
IBM	0.246
Apple	0.452
Microsoft	3.126
Nike	2.619
ADM	0.953
S&P500	1.326
Small Firm	1.912
Govt Bond	1.057
DP(start)	2.887

IV. How to calculate the variance and standard deviation of return.

A. Formula: If there are K possible states: s1,s2,...,,sK,

$\sigma^2[R_{Asset}]$ = Variance of Return on the Asset

$= pr(s1)\ \{R_{Asset}(s1)-E[R_{Asset}]\}^2 + pr(s2)\ \{R_{Asset}(s2)-E[R_{Asset}\}^2 + ...$
$+ pr(sK)\ \{R_{Asset}(sK)-E[R_{Asset}]\}^2$

where R_{Asset} (s) is the return on the Asset in state s; and
pr(s) is the probability of state s.

$\sigma[R_{Asset}]$ = Standard Deviation of Return on the Asset

$= \sqrt{\sigma^2[R_{Asset}]}$

B. Example 1:

1. Formula: There are 3 states. So:

$\sigma^2[R_{Asset}]$ = Variance of Return on the Asset

$= pr(BM)\ \{R_{Asset}(BM)-E[R_{Asset}]\}^2 + pr(RC)\ \{R_{Asset}(RC)-E[R_{Asset}\}^2$
$+ pr(DP)\ \{R_{Asset}(DP)-E[R_{Asset}]\}^2$

2. Calculations:

$\sigma^2[R_{Ford}] = 0.50 \times (25-9.6)(25-9.6) + 0.30 \times (-3-9.6)(-3-9.6) + 0.20 \times (-10-9.6)(-10-9.6)$
$= 118.58 + 47.628 + 76.832 = 243.04.$
$\sigma[R_{Ford}] = \sqrt{243.04} = 15.5897\%.$

$\sigma^2[R_{GM}] = 0.50 \times (22-7.8)(22-7.8) + 0.30 \times (2-7.8)(2-7.8) + 0.20 \times (-19-7.8)(-19-7.8)$
$= 100.82 + 10.092 + 143.648 = 254.56.$
$\sigma[R_{GM}] = \sqrt{254.56} = 15.9549\%.$

$\sigma^2[R_{Reebok}] = 0.50 \times (9-7.8)(9-7.8) + 0.30 \times (9-7.8)(9-7.8) + 0.20 \times (3-7.8)(3-7.8)$
$= 0.72 + 0.432 + 4.608 = 5.76.$
$\sigma[R_{Reebok}] = \sqrt{5.76} = 2.4\%.$

$\sigma^2[R_{T\text{-}bill}] = 0.50 \times (5-5)(5-5) + 0.30 \times (5-5)(5-5) + 0.20 \times (5-5)(5-5)$
$= 0 + 0 + 0 = 0.$
$\sigma[R_{T\text{-}bill}] = \sqrt{0} = 0\%.$

C. Example 2:

Asset	σ [R]
IBM	8.004
Apple	13.050
Microsoft	8.203
Nike	9.265
ADM	6.712
S&P 500	2.886
Small Firm	3.711
Govt Bond	2.272
DP(start)	0.279

V. How to calculate the covariance and correlation between two returns.

A. Formula: If there are K possible states: s1,s2,...,,sK,

$\sigma[R_{A1},R_{A2}]$ = Covariance of the Return on Asset 1with the Return on Asset 2
= pr(s1) $\{R_{A1}(s1)-E[R_{A1}]\}$ $\{R_{A2}(s1)-E[R_{A2}]\}$
+ pr(s2) $\{R_{A1}(s2)-E[R_{A1}]\}$ $\{R_{A2}(s2)-E[R_{A2}]\}$ + ...
+ pr(sK) $\{R_{A1}(sK)-E[R_{A1}]\}$ $\{R_{A2}(sK)-E[R_{A2}]\}$

where R_{A1} (s) is the return on Asset 1 in state s;
R_{A2} (s) is the return on Asset 2 in state s; and
pr(s) is the probability of state s.

$\rho[R_{A1},R_{A2}]$ = Correlation of the Return on Asset 1with the Return on Asset 2

$$= \frac{\sigma[R_{A1},R_{A2}]}{\sigma[R_{A1}]\ \sigma[R_{A2}]}$$

B. Example 1:

1. Formula: There are 3 states. So:

$\sigma[R_{A1},R_{A2}]$ = Covariance of the Return on Asset 1with the Return on Asset 2
= pr(BM) $\{R_{A1}(BM)-E[R_{A1}]\}$ $\{R_{A2}(BM)-E[R_{A2}]\}$
+ pr(RC) $\{R_{A1}(RC)-E[R_{A1}]\}$ $\{R_{A2}(RC)-E[R_{A2}]\}$
+ pr(DP) $\{R_{A1}(DP)-E[R_{A1}]\}$ $\{R_{A2}(DP)-E[R_{A2}]\}$

2. Calculations:

$\sigma[R_{Ford},R_{GM}]$ = 0.50 x (25-9.6)(22-7.8) + 0.30 x (-3-9.6)(2-7.8) + 0.20 x (-10-9.6)(-19-7.8)
= 109.34 + 21.924 + 105.056 = 236.32.
$\rho[R_{Ford},R_{GM}]$ = 236.32/(15.5897x15.9549) = 0.95009.

$\sigma[R_{Ford},R_{Reebok}]$ = 0.50 x (25-9.6)(9-7.8) + 0.30 x (-3-9.6)(9-7.8) + 0.20 x (-10-9.6)(3-7.8)
= 9.24 - 4.536 + 18.816 = 23.52.
$\rho[R_{Ford},R_{Reebok}]$ = 23.52/(15.5897x2.4) = 0.6286.

$\sigma[R_{GM},R_{Reebok}]$ = 0.50 x (22-7.8)(9-7.8) + 0.30 x (2-7.8)(9-7.8) + 0.20 x (-19-7.8)(3-7.8)
= 8.52 - 2.088 + 128.64 = 32.16.
$\rho[R_{GM},R_{Reebok}]$ = 32.16/(15.9549x2.4) = 0.8398.

$\sigma[R_{Ford},R_{T\text{-}bill}]$ = 0.50 x (25-9.6)(5-5) + 0.30 x (-3-9.6)(5-5) + 0.20 x (-10-9.6)(5-5)
= 48 - 48 = 0.

C. Example 2:

$\sigma[R_i,R_j]$	i=								$\sigma[DP,R_j]$
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond	DP(start)
IBM	64.060	23.590	21.020	10.720	-5.017	5.274	1.338	-1.892	-0.230
Apple	23.590	170.300	47.160	38.790	21.250	13.570	22.890	-0.778	0.725
Microsoft	21.020	47.160	67.290	23.670	8.861	8.814	12.030	-0.134	0.680
Nike	10.720	38.790	23.670	85.830	17.480	9.952	8.951	3.322	-0.119
ADM	-5.017	21.250	8.861	17.480	45.050	8.235	4.521	5.778	0.017
S&P 500	5.274	13.570	8.814	9.952	8.235	8.328	6.647	3.150	0.081
Small Firm	1.338	22.890	12.030	8.951	4.521	6.647	13.770	0.824	0.303
Govt Bond	-1.892	-0.778	-0.134	3.322	5.778	3.150	0.824	5.162	-0.031
$\sigma[R_i,DP]$	-0.230	0.725	0.680	-0.119	0.017	0.081	0.303	-0.031	0.078

$\rho[R_i,R_j]$	i=								$\rho[DP,R_j]$
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond	DP(start)
IBM	1.000	0.226	0.320	0.145	-0.093	0.228	0.045	-0.104	-0.103
Apple	0.226	1.000	0.441	0.321	0.243	0.360	0.473	-0.026	0.199
Microsoft	0.320	0.441	1.000	0.312	0.161	0.372	0.395	-0.007	0.297
Nike	0.145	0.321	0.312	1.000	0.281	0.372	0.260	0.158	-0.046
ADM	-0.093	0.243	0.161	0.281	1.000	0.425	0.182	0.379	0.009
S&P 500	0.228	0.360	0.372	0.372	0.425	1.000	0.621	0.480	0.101
Small Firm	0.045	0.473	0.395	0.260	0.182	0.621	1.000	0.098	0.293
Govt Bond	-0.104	-0.026	-0.007	0.158	0.379	0.480	0.098	1.000	-0.050
$\rho[R_i,DP]$	-0.103	0.199	0.297	-0.046	0.009	0.101	0.293	-0.050	1.000

D. Points to Note:

1. Covariance of Return(Asset1, Asset1) = Variance of Return(Asset).
2. Correlation of Return(Asset1, Asset1) = 1.

VI. Regression

A. Expectation and Regression.

1. Consider the following way of writing the return on asset i:

$R_i = \mu_i + e_i$;

where μ_i is a constant. Both R_i and e_i are random variables.

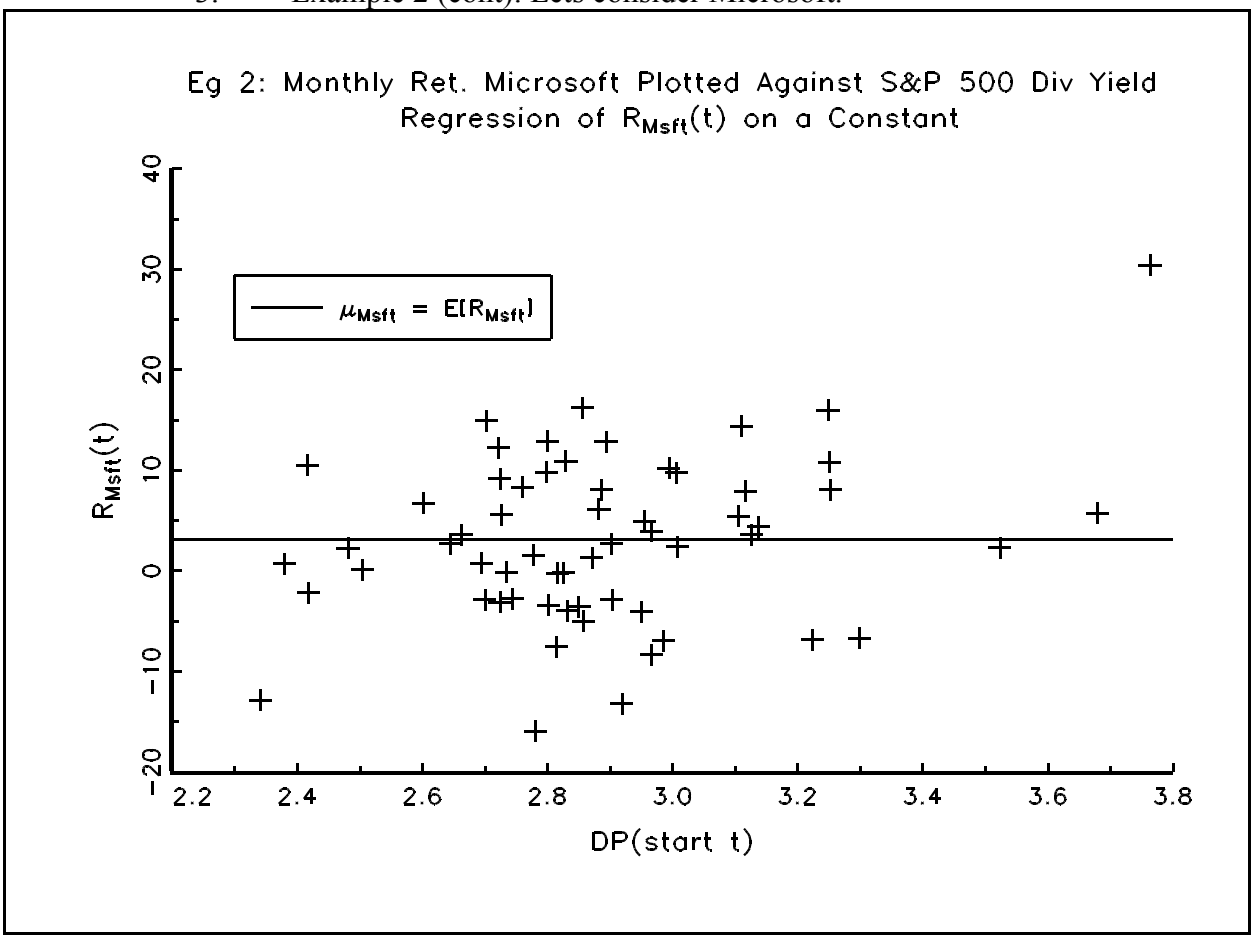
a. Can think of μ_i as a prediction of the return on asset i given that you have no information.

b. Suppose we want to choose μ_i to minimize the magnitude of e_i.

c. Taking $\mu_i = E[R_i]$ gives the e_i with the smallest $E[e_i^2]$. Also $E[e_i] = 0$.

2. So $E[R_i]$ is the constant that minimises the mean squared prediction error.

3. Example 2 (cont): Lets consider Microsoft.



B. One Independent Variable.

1. Suppose now that the return prediction can be a linear function of a variable known at the start of the period (for example, dividend yield on the S&P 500 portfolio DP): $\mu_{i,DP} + \varphi_{i,DP}$ DP where $\mu_{i,DP}$ and $\varphi_{i,DP}$ are constants.
2. Consider now the question of finding the values of $\mu_{i,DP}$ and $\varphi_{i,DP}$ which minimize the mean squared prediction error $E[e_{i,DP}^2]$:

$$R_i = \mu_{i,DP} + \varphi_{i,DP} DP + e_{i,DP}$$

3. The $\mu_{i,DP}$ and $\varphi_{i,DP}$ which minimize $E[e_{i,DP}^2]$ can be shown to satisfy:

$$\varphi_{i,DP} = \frac{\sigma[R_i, DP]}{\sigma[DP]^2}; \quad and, \quad \mu_{i,DP} = E[R_i] - \varphi_{i,DP} E[DP] .$$

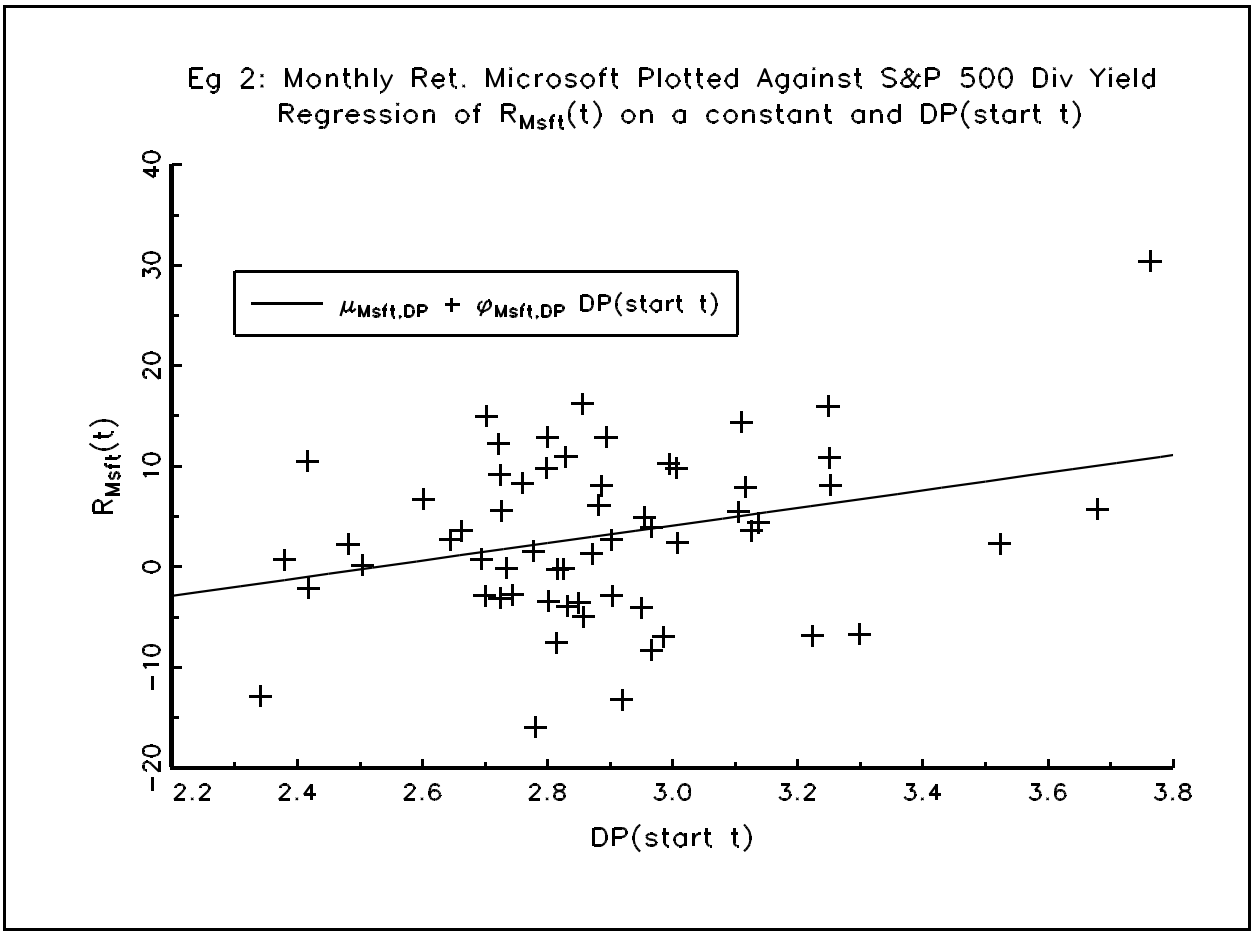
Further, it can be shown $E[e_{i,DP}] = 0$ and $\sigma[DP, e_{i,DP}] = 0$. For most random variables, it is possible to find $\mu_{i,DP}$ and $\varphi_{i,DP}$ which minimize $E[e_{i,DP}^2]$. These are known as regression coefficients.

4. Note that $E[e_{i,DP}^2] \leq E[e_i^2]$. So the mean squared prediction error is reduced by using DP to help predict asset return.
5. Can (roughly) think of $\mu_{i,DP} + \varphi_{i,DP}$ DP as the expected return on asset i given that you know DP, and $E[e_{i,DP}^2]$ as the return variance given that you know DP.

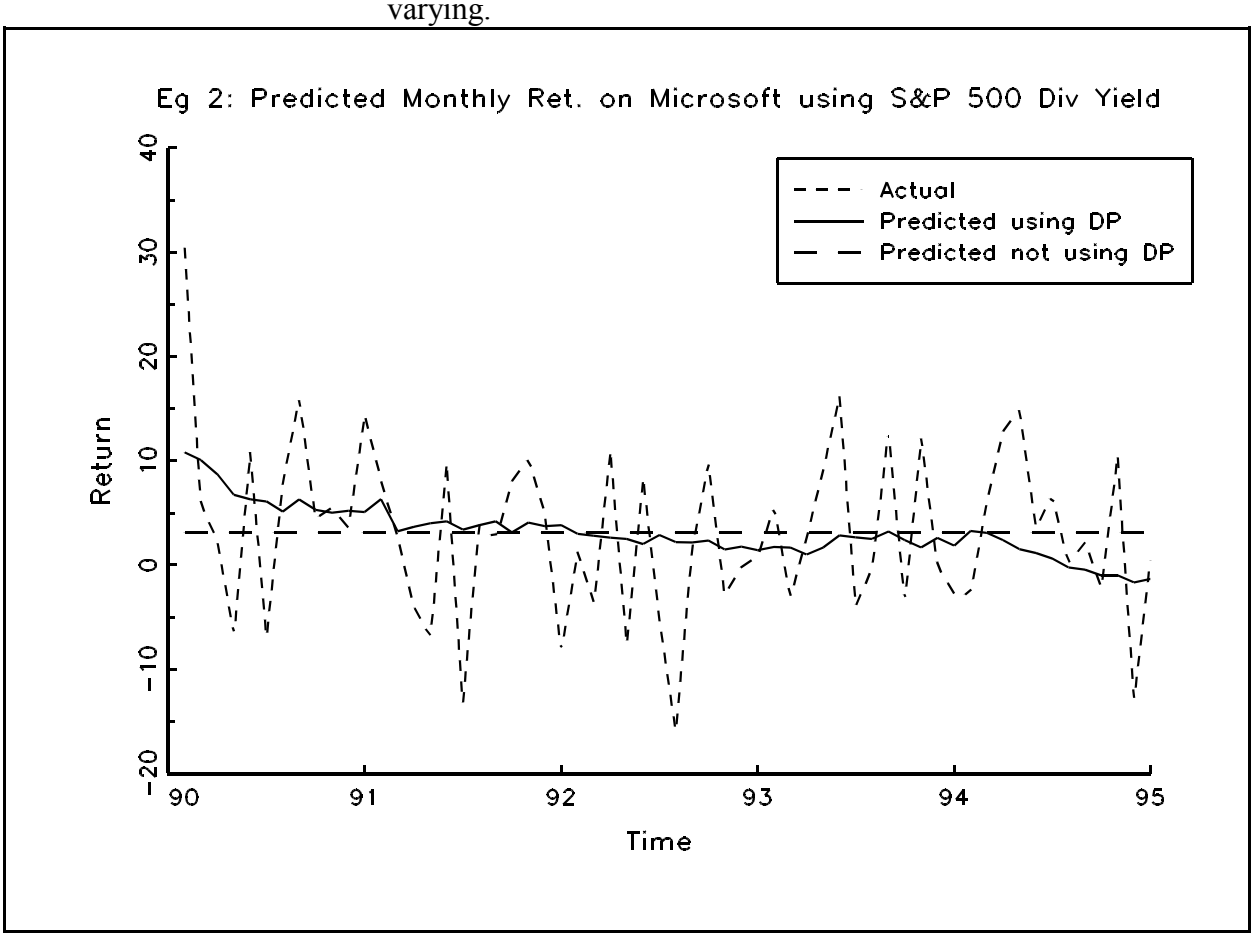
6. Example 2 (cont):
   a. Regress the 8 assets on the S&P 500 Dividend Yield and a constant using the historical data:

Asset	$\mu_{i,DP}$	$\varphi_{i,DP}$
IBM	8.770	-2.952
Apple	-26.500	9.333
Microsoft	-22.140	8.749
Nike	7.028	-1.527
ADM	0.316	0.221
S&P 500	-1.700	1.048
Small Firm	-9.330	3.893
Govt Bond	2.224	-0.404

   b. Graphically, regress Microsoft on a constant and the S&P 500 Dividend Yield.



c. Note that the predicted return on Microsoft moves as the S&P 500 Dividend Yield moves; so Microsoft's expected return is time-varying.



C. More generally, if Y and X_1, X_2, ... ,X_K are random variables, there exists a set of regression coefficients φ_0, φ_1, φ_2, ... φ_K such that the random error e from

$$Y = \varphi_0 + \varphi_1 X_1 + \varphi_2 X_2 + ... + \varphi_K X_K + e$$

satisfies:

1. $E[e] = 0$;
2. $\sigma[X_k, e] = 0$ for $k = 1,2,...,K$; and,
3. the regression coefficients φ_0, φ_1, φ_2, ... φ_K minimize $E[e^2]$.

VII. Determining Expected Returns and Return Covariances for a Given Month at the Start of that Month

A. Example 2: It is the end of January 1997 and the investor needs expected February returns and covariances between the February returns for the 8 assets discussed above.

1. Ignore Dividend Yield Information (Unconditional).

a. May ignore DP because you are skeptical that it has predictive ability for stocks and bonds.

b. Expected February return: Use the $E[R_i]$s calculated above from the historical distribution.

c. Standard Deviations: Use the $\sigma[R_i]$s calculated above from the historical distribution.

d. Covariances: Use the $\sigma[R_i,R_j]$s calculated above from the historical distribution.

2. Use Dividend Yield Information (Conditional).
   a. DP(start 2/97) = 2.0087%.
   b. Expected February return: For a given asset i, the expected February return is given by: $\mu_{i,DP} + \varphi_{i,DP}$ DP(start 2/97) where the regression coefficients are those obtained using the historical data:

	Unconditional	Conditional
Asset i	E [R_i]	$\mu_{i,DP} + \varphi_{i,DP}$ DP(start 2/97)
IBM	0.246	2.840
Apple	0.452	-7.750
Microsoft	3.126	-4.562
Nike	2.619	3.961
ADM	0.953	0.759
S&P500	1.326	0.405
Small Firm	1.912	-1.509
Govt Bond	1.057	1.412

   c. Standard Deviations: Calculate standard deviations for the residuals ($e_{i,DP}$ s) of the DP regressions (run using the historical data):

	Unconditional	Conditional
Asset i	σ[R_i]	σ[$e_{i,DP}$]
IBM	8.004	7.961
Apple	13.050	12.790
Microsoft	8.203	7.832
Nike	9.265	9.255
ADM	6.712	6.711
S&P 500	2.886	2.871
Small Firm	3.711	3.549
Govt Bond	2.272	2.269

d. Covariances: Calculate covariances for the residuals ($e_{i,DP}$ s) of the DP regressions (run using the historical data):

Conditional (using DP)								
$\sigma[e_{i,DP},e_{j,DP}]$	i=							
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond
IBM	63.380	25.730	23.030	10.370	-4.967	5.515	2.232	-1.985
Apple	25.730	163.600	40.810	39.900	21.090	12.810	20.060	-0.485
Microsoft	23.030	40.810	61.340	24.710	8.711	8.102	9.383	0.141
Nike	10.370	39.900	24.710	85.650	17.500	10.080	9.413	3.274
ADM	-4.967	21.090	8.711	17.500	45.040	8.217	4.454	5.784
S&P 500	5.515	12.810	8.102	10.080	8.217	8.243	6.330	3.183
Small Firm	2.232	20.060	9.383	9.413	4.454	6.330	12.590	0.946
Govt Bond	-1.985	-0.485	0.141	3.274	5.784	3.183	0.946	5.150

Unconditional (not using DP)								
$\sigma[R_i,R_j]$	i=							
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond
IBM	64.060	23.590	21.020	10.720	-5.017	5.274	1.338	-1.892
Apple	23.590	170.300	47.160	38.790	21.250	13.570	22.890	-0.778
Microsoft	21.020	47.160	67.290	23.670	8.861	8.814	12.030	-0.134
Nike	10.720	38.790	23.670	85.830	17.480	9.952	8.951	3.322
ADM	-5.017	21.250	8.861	17.480	45.050	8.235	4.521	5.778
S&P 500	5.274	13.570	8.814	9.952	8.235	8.328	6.647	3.150
Small Firm	1.338	22.890	12.030	8.951	4.521	6.647	13.770	0.824
Govt Bond	-1.892	-0.778	-0.134	3.322	5.778	3.150	0.824	5.162

Conditional (using DP)								
$\rho[e_{i,DP},e_{j,DP}]$	i=							
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond
IBM	1.000	0.253	0.369	0.141	-0.093	0.241	0.079	-0.110
Apple	0.253	1.000	0.407	0.337	0.246	0.349	0.442	-0.017
Microsoft	0.369	0.407	1.000	0.341	0.166	0.360	0.338	0.008
Nike	0.141	0.337	0.341	1.000	0.282	0.379	0.287	0.156
ADM	-0.093	0.246	0.166	0.282	1.000	0.426	0.187	0.380
S&P 500	0.241	0.349	0.360	0.379	0.426	1.000	0.621	0.489
Small Firm	0.079	0.442	0.338	0.287	0.187	0.621	1.000	0.118
Govt Bond	-0.110	-0.017	0.008	0.156	0.380	0.489	0.118	1.000

Unconditional (not using DP)								
$\rho[R_i,R_j]$	i=							
j=	IBM	Apple	Microsoft	Nike	ADM	S&P 500	Small Firm	Govt Bond
IBM	1.000	0.226	0.320	0.145	-0.093	0.228	0.045	-0.104
Apple	0.226	1.000	0.441	0.321	0.243	0.360	0.473	-0.026
Microsoft	0.320	0.441	1.000	0.312	0.161	0.372	0.395	-0.007
Nike	0.145	0.321	0.312	1.000	0.281	0.372	0.260	0.158
ADM	-0.093	0.243	0.161	0.281	1.000	0.425	0.182	0.379
S&P 500	0.228	0.360	0.372	0.372	0.425	1.000	0.621	0.480
Small Firm	0.045	0.473	0.395	0.260	0.182	0.621	1.000	0.098
Govt Bond	-0.104	-0.026	-0.007	0.158	0.379	0.480	0.098	1.000

3. Also need a riskless rate for February 1997.
   a. Return on a U.S. T-bill purchased 1/31/97 and maturing on 2/27/97 (based on an asked yield quote of 4.91% and 24 days to maturity in the WSJ) is 0.323%.

Monthly Return Data for Example 2

Date	Prob	Monthly Return (%)								S&P500 Div Yld (start)
		IBM	Apple	Micro-soft	Nike	ADM	S&P 500	Small Firm	Govt Bonds	
9101	0.01667	12.168	29.070	30.399	18.634	-10.330	4.344	8.410	1.304	3.763
9102	0.01667	2.533	3.369	5.733	0.524	9.203	7.143	11.130	0.304	3.680
9103	0.01667	-11.553	18.777	2.289	-3.615	-1.124	2.419	6.800	0.381	3.523
9104	0.01667	-9.550	-19.118	-6.714	2.981	3.977	0.235	0.340	1.403	3.299
9105	0.01667	4.209	-14.327	10.859	-16.316	-1.530	4.302	3.340	0.005	3.251
9106	0.01667	-8.481	-11.702	-6.891	-8.767	5.000	-4.578	-4.850	-0.632	3.224
9107	0.01667	4.247	11.446	7.890	20.761	5.820	4.657	4.070	1.575	3.117
9108	0.01667	-3.126	14.854	15.986	14.327	8.250	2.362	2.610	3.404	3.249
9109	0.01667	6.968	-6.604	4.399	8.802	-0.485	-1.674	0.320	3.035	3.137
9110	0.01667	-5.187	4.040	5.478	2.079	6.829	1.339	3.170	0.544	3.105
9111	0.01667	-4.621	-1.223	3.595	6.561	0.091	-4.016	-2.760	0.820	3.126
9112	0.01667	-3.784	11.084	14.396	23.185	21.005	11.417	6.010	5.808	3.110
9201	0.01667	1.124	14.856	8.090	1.727	-10.189	-1.864	11.280	-3.241	3.252
9202	0.01667	-2.128	4.432	2.703	-0.849	-1.177	1.297	4.520	0.511	2.902
9203	0.01667	-3.885	-13.704	-4.049	-7.500	-10.213	-1.943	-2.490	-0.935	2.950
9204	0.01667	8.683	3.219	-6.962	-7.050	-8.057	2.932	-4.030	0.159	2.986
9205	0.01667	1.333	-0.624	9.751	-7.385	6.289	0.489	-0.140	2.426	3.007
9206	0.01667	7.851	-19.464	-13.223	7.371	-8.252	-1.487	-5.200	2.000	2.919
9207	0.01667	-3.193	-2.604	3.929	15.493	15.344	4.084	3.700	3.978	2.966
9208	0.01667	-7.298	-1.348	2.406	0.871	-5.505	-2.045	-2.280	0.669	3.008
9209	0.01667	-6.782	-1.902	8.054	10.052	1.531	1.176	1.310	1.855	2.887
9210	0.01667	-17.183	16.344	10.249	-2.044	6.030	0.343	2.590	-1.981	2.994
9211	0.01667	3.865	9.752	4.930	10.754	3.412	3.396	8.850	0.100	2.955
9212	0.01667	-26.191	3.913	-8.322	-3.536	-2.752	1.227	4.410	2.461	2.967
9301	0.01667	2.233	-0.418	1.318	-1.657	-0.943	0.835	5.430	2.804	2.871
9302	0.01667	6.631	-10.723	-3.613	-11.639	1.048	1.362	-1.800	3.539	2.850
9303	0.01667	-6.437	-2.830	10.945	6.517	-2.830	2.109	3.000	0.213	2.829
9304	0.01667	-4.423	-0.485	-7.568	-10.930	-2.913	-2.417	-3.060	0.719	2.815
9305	0.01667	9.594	10.722	8.333	6.227	-4.400	2.667	3.420	0.466	2.760
9306	0.01667	-6.398	-30.243	-4.993	-23.690	-1.047	0.292	-0.380	4.487	2.857
9307	0.01667	-9.873	-29.747	-15.909	2.041	-0.529	-0.403	1.660	1.915	2.781
9308	0.01667	3.371	-4.072	1.520	-4.222	2.872	3.786	3.390	4.338	2.778
9309	0.01667	-8.197	-11.792	9.817	-16.102	-2.717	-0.767	3.160	0.052	2.798
9310	0.01667	9.524	31.551	-2.879	7.500	2.793	2.067	4.710	0.964	2.700
9311	0.01667	17.663	2.829	-0.156	-1.034	-3.152	-0.949	-1.750	-2.588	2.734
9312	0.01667	4.872	-7.143	0.781	-2.977	2.247	1.209	1.940	0.197	2.694

Date	Prob	Return (%)								S&P500 Div Yld (start)
		IBM	Apple	Micro-soft	Nike	ADM	S&P 500	Small Firm	Govt Bonds	
9401	0.01667	0.000	11.966	5.581	8.108	16.662	3.397	6.180	2.573	2.727
9402	0.01667	-5.973	11.817	-3.084	3.500	-3.327	-2.712	-0.230	-4.496	2.724
9403	0.01667	3.310	-8.904	2.727	2.802	-6.331	-4.354	-4.460	-3.950	2.645
9404	0.01667	5.263	-9.774	9.145	1.189	-4.199	1.283	0.600	-1.504	2.724
9405	0.01667	10.000	-2.100	16.216	10.013	5.041	1.637	-0.120	-0.824	2.856
9406	0.01667	-6.746	-9.402	-3.953	1.610	-3.133	-2.449	-2.620	-1.005	2.832
9407	0.01667	5.319	27.123	-0.242	2.929	5.323	3.281	1.840	3.633	2.816
9408	0.01667	11.111	7.777	12.864	4.675	9.955	4.093	3.370	-0.857	2.894
9409	0.01667	1.642	-6.908	-3.441	-7.650	0.932	-2.441	1.050	-3.307	2.801
9410	0.01667	7.002	28.200	12.249	2.743	10.242	2.239	1.150	-0.245	2.722
9411	0.01667	-4.698	-13.470	-0.198	4.928	-3.459	-3.637	-3.260	0.661	2.826
9412	0.01667	3.887	4.027	-2.783	17.221	11.998	1.480	0.020	1.607	2.743
9501	0.01667	-1.871	4.194	-2.863	-4.858	-2.302	2.592	2.830	2.729	2.903
9502	0.01667	4.672	-1.882	6.096	1.239	-6.208	3.893	2.520	2.866	2.882
9503	0.01667	9.143	-10.759	12.905	4.688	-1.335	2.946	1.450	0.915	2.800
9504	0.01667	15.220	8.511	14.930	2.160	-2.029	2.943	3.520	1.694	2.702
9505	0.01667	-1.458	8.967	3.596	2.950	1.519	3.989	2.980	7.903	2.663
9506	0.01667	3.226	11.742	6.719	6.808	0.681	2.321	5.680	1.388	2.602
9507	0.01667	13.417	-3.101	0.133	7.595	-11.443	3.314	6.450	-1.677	2.505
9508	0.01667	-4.822	-4.178	2.210	2.478	6.009	0.250	3.580	2.361	2.482
9509	0.01667	-8.590	-13.372	-2.162	20.244	-7.517	4.218	1.950	1.748	2.418
9510	0.01667	2.910	-2.523	10.497	1.926	4.031	-0.357	-4.870	2.941	2.417
9511	0.01667	-0.380	5.343	-12.870	2.419	8.125	4.386	1.920	2.493	2.341
9512	0.01667	-5.433	-16.391	0.712	20.310	4.348	1.926	2.390	2.719	2.379