**Lecture 20: Bond Portfolio Management.**

I. Reading.
   A. BKM, Chapter 16, Sections 16.1 and 16.2.

II. Risks associated with Fixed Income Investments.
   A. Reinvestment Risk.
      1. If an individual has a particular time horizon T and holds an instrument with a fixed cash flow received prior to T, then the investor faces uncertainty about what yields will prevail at the time of the cash flow. This uncertainty is known as reinvestment risk.
      2. Example: Suppose an investor has to meet a obligation of $5M in two years time. If she buys a two year coupon bond to meet this obligation, there is uncertainty about the rate at which the coupons on the bond can be invested. This uncertainty is an example of reinvestment risk.

   B. Liquidation Risk.
      1. If an individual has a particular time horizon T and holds an instrument which generates cash flows that are received after T, then the investor faces uncertainty about the price of the instrument at time T. This uncertainty is known as liquidation risk.
      2. Example: Suppose an investor has to meet a obligation of $5M in two years time. If she buys a five year discount bond to meet this obligation, there is uncertainty about the price at which this bond will sell in two years time. This uncertainty is an example of liquidation risk.

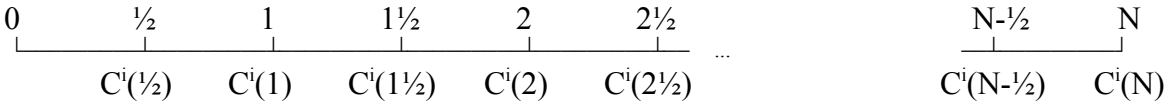
III. Duration.

A. Definition.

1. Macaulay Duration.

a. Let 1 period=1 year.

b. Consider a fixed income instrument i which generates the following stream of certain cash flows:




Let $y^i(0)$ be the YTM at time 0 on the instrument expressed as an APR with semi-annual compounding.

c. Macaulay duration is defined as

$$D^i(0) = \tfrac{1}{2}\, k^i(\tfrac{1}{2}) + 1\, k^i(1) + 1\tfrac{1}{2}\, k^i(1\tfrac{1}{2}) + \ldots + N\, k^i(N)$$

$$\text{where} \quad k^i(t) = \frac{1}{P^i(0)} \frac{C^i(t)}{[1+y^i(0)/2]^{2t}} .$$

d. Note that the $k^i(t)$s sum to 1:

$$1 = k^i(\tfrac{1}{2}) + k^i(1) + k^i(1\tfrac{1}{2}) + \ldots + k^i(N) .$$

2. "Modified" Duration.

a. "Modified" duration $D^{*i}(0)$ is defined to be Macaulay duration divided by $[1+y^i(0)/2]$:

$$D^{*i}(0) = D^i(0) /[1+y^i(0)/2].$$

b. "Modified" duration is often used in the industry.

3. Example. 2/15/97. The 3 Feb 98 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.
   a. Its k's can be calculated:

$k^{3\text{ Feb }98}(½)$ = (1.5/1.03)/97.1298 = 1.4563/97.1298 = 0.0150.
$k^{3\text{ Feb }98}(1)$ = $(101.5/1.03^2)$/97.1298 = 95.6735/97.1298 = 0.9850.

   b. Its duration can be calculated:

$D^{3\text{ Feb }98}(0)$ = ½ x $k^{3\text{ Feb }98}(½)$ + 1 x $k^{3\text{ Feb }98}(1)$
= ½ x 0.0150 + 1 x 0.9850 = 0.9925 years.

   c. Its modified duration can be calculated:

$D^{*\ 3\text{ Feb }98}(0)$ = 0.9925 /(1+0.03) = 0.9636.

| Instrument | YTM(2/15/97) | Duration(2/15/97) |
|---|---|---|
| Feb 98 Strip | 6% | 1.0000 |
| 3 Feb 98 Bond | 6% | 0.9925 |
| 10 Feb 98 Bond | 6% | 0.9766 |
| 40% of 3 Feb 98 and<br>60% of 10 Feb 98 | 6%<br>6% | 0.9925 x 0.4 +<br>0.9766 x 0.6 = 0.98296 |

B. Duration can be interpreted as the effective maturity of the instrument.

1. If the instrument is a zero coupon bond, then its duration is equal to the time until maturity.
2. For an instrument with cash flows prior to maturity, its duration is less than the time until maturity.
3. Holding maturity and YTM constant, the larger the earlier payments relative to the later payments then the shorter the duration.
4. Example: 2/15/97. Suppose the 10 Feb 98 coupon bond has a price of 103.8270 and a YTM expressed as an APR with semi-annual compounding of 6%.

a. Its k's can be calculated:

$k^{10\text{ Feb }98}(½)$ $= (5/1.03)/103.8270 = 4.8544/103.8270 = 0.0468.$

$k^{10\text{ Feb }98}(1)$ $= (105/1.03^2)/103.8270 = 98.9726/103.8270 = 0.9532.$

b. Its duration can be calculated:

$D^{10\text{ Feb }98}(0)$ $= ½ \text{ x } k^{10\text{ Feb }98}(½) + 1 \text{ x } k^{10\text{ Feb }98}(1)$

$= ½ \text{ x } 0.0468 + 1 \text{ x } 0.9532 = 0.9766$ years.

c. So the 10 Feb 98 bond with larger earlier cash flows relative to later cash flows has a shorter duration than the 3 Feb 98 bond.

C. Duration of a Portfolio.

1. If a portfolio C has $\omega^{A,C}$ invested in asset A at time 0 and $\omega^{B,C}$ invested in asset B at time 0 and the YTMs of A and B are the same, the duration of C is given by:

$$D^C(0) = \omega^{A,C} D^A(0) + \omega^{B,C} D^B(0) .$$

2. Example (cont): 2/15/97. Suppose Tom forms a portfolio on 2/15/97 with 40% in 3 Feb 98 coupon bonds and the rest in 10 Feb 98 coupon bonds.

a. The portfolio's duration can be calculated:

$D^P(0)$ $= \omega^{3\text{ Feb }98,P} D^{3\text{ Feb }98}(0) + \omega^{10\text{ Feb }98,P} D^{10\text{ Feb }98}(0)$

$= 0.4 \text{ x } 0.9925 + (1-0.4) \text{ x } 0.9766 = 0.98296.$

D. Duration as a measure of yield sensitivity.

1. The price of bond i described above is given by:

$$P^i(0) = \frac{C^i(½)}{[1+y^i(0)/2]^1} + \frac{C^i(1)}{[1+y^i(0)/2]^2} + \frac{C^i(1½)}{[1+y^i(0)/2]^3} + \ldots + \frac{C^i(N)}{[1+y^i(0)/2]^{2N}} .$$

2. To assess the impact of the price of bond i to a shift in $y^i(0)$, differentiate the above expression with respect to $y^i(0)$:

$$\frac{d\,P^i(0)}{d\,y^i(0)} = -\,½\,\frac{C^i(½)}{[1+y^i(0)/2]^2} - 1\,\frac{C^i(1)}{[1+y^i(0)/2]^3} - 1½\,\frac{C^i(1½)}{[1+y^i(0)/2]^4} - \ldots - N\,\frac{C^i(N)}{[1+y^i(0)/2]^{2N+1}}$$

3. Thus, bond i's modified Macaulay duration is related to the sensitivity of its price to shifts in $y^i(0)$ as follows:

$$\frac{d\,P^i(0)}{d\,y^i(0)} = -P^i(0)\,D^{*i}(0) .$$

4. Thus, bond i's modified Macaulay duration measures its price sensitivity to a change in its yield (where price sensitivity is measured by the percentage change in the price).

5. In particular, a small change in $y^i(0)$, $\Delta y^i(0)$, causes a change in $P^i(0)$, $\Delta P^i(0)$, which satisfies:

$$\Delta\,P^i(0) \approx -P^i(0)\,D^{*i}(0)\,\Delta y^i(0) .$$

6. Example (cont): 2/15/97. The 3 Feb 98 note has a price of 97.1298 and a YTM expressed as an APR with semi-annual compounding of 6%.
   a. Suppose the YTM on the 3 Feb 98 bond increases by 1% to 7%. The bond's price becomes

$P^{3\ Feb\ 98}(0) = 1.5/(1+0.07/2) + 101.5/(1+0.7/2)^2 = 96.2006.$

which means a price decline of 0.9292.

   b. Suppose the YTM on the 3 Feb 98 bond decreases by 1% to 5%. The bond's price becomes

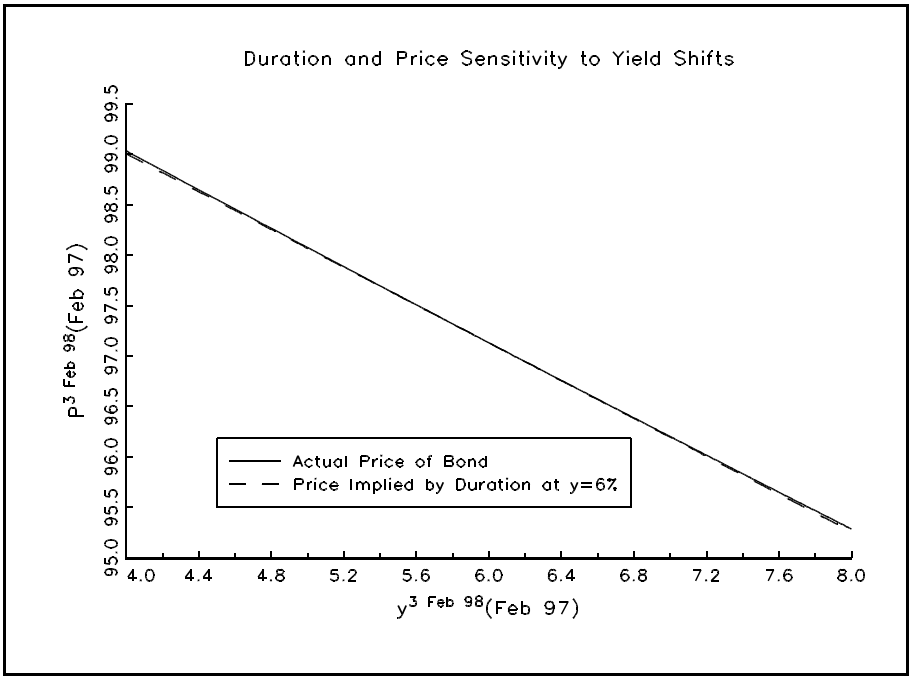
$P^{3\ Feb\ 98}(0) = 1.5/(1+0.05/2) + 101.5/(1+0.05/2)^2 = 98.0725.$

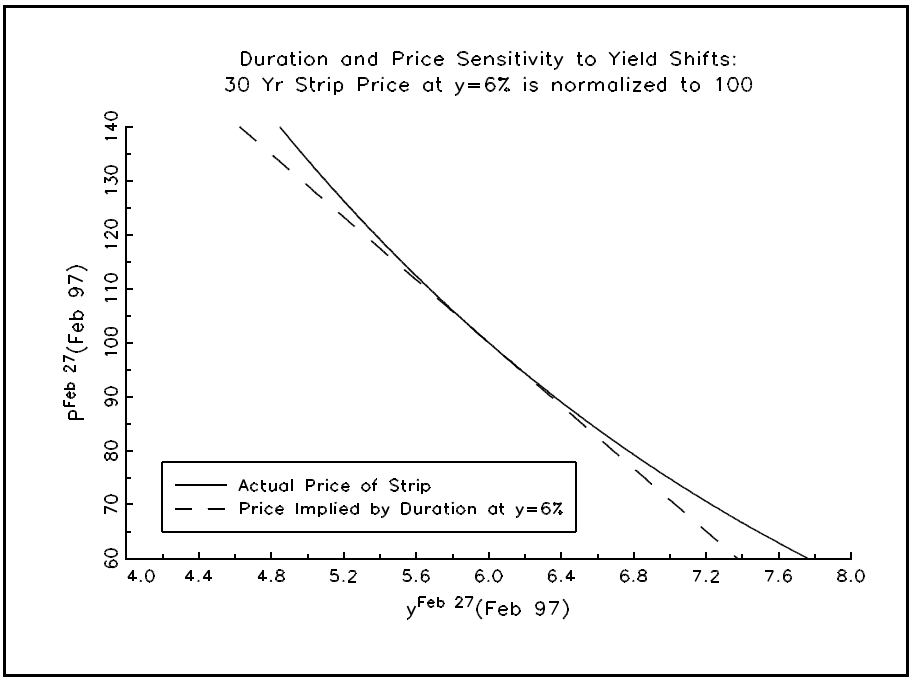
which implies a price increase of 0.9427.

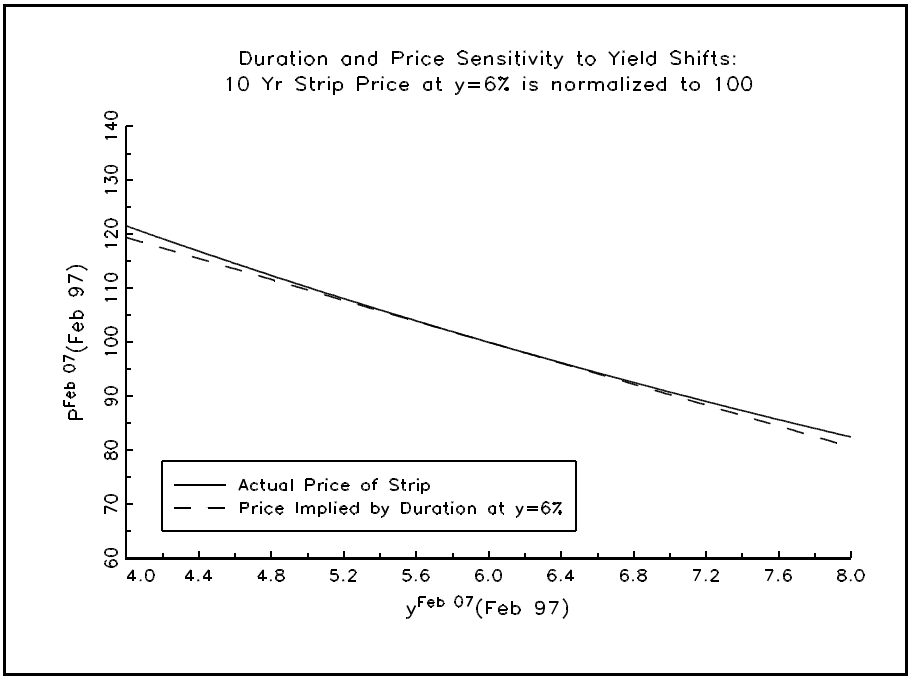
   c. Using the formula involving modified duration above:

YTM increase: $\Delta P^{3\ Feb\ 98}(0) = -97.1298 \times 0.9636 \times 0.01 = -0.9359.$
YTM decrease: $\Delta P^{3\ Feb\ 98}(0) = -97.1298 \times 0.9636 \times -0.01 = 0.9359.$

   d. So the duration based approximation overstates the price decline when the YTM increases and understates the price increase when the YTM decreases. It can be shown that this is generally true.




Duration and Price Sensitivity to Yield Shifts:
30 Yr Strip Price at y=6% is normalized to 100
pFeb 27(Feb 97)
yFeb 27(Feb 97)
Actual Price of Strip
Price Implied by Duration at y=6%
60
70
80
90
100
110
120
130
140
4.0
4.4
4.8
5.2
5.6
6.0
6.4
6.8
7.2
7.6
8.0


Duration and Price Sensitivity to Yield Shifts:
10 Yr Strip Price at y=6% is normalized to 100
pFeb 07(Feb 97)
yFeb 07(Feb 97)
Actual Price of Strip
Price Implied by Duration at y=6%
60
70
80
90
100
110
120
130
140
4.0
4.4
4.8
5.2
5.6
6.0
6.4
6.8
7.2
7.6
8.0

| 3 Feb 98 | | | YTM = 6% | | | |
|---|---|---|---|---|---|---|
| YTM | P(0) | Actual ΔP(0) | P(0) | D*(0) | ΔYTM | Approx ΔP(0) = -P(0)D*(0)ΔYTM |
| 7% | 96.2006 | -0.9292 | 97.1298 | 0.9636 | +0.01 | -0.9359 |
| 6% | 97.1298 | 0 | | | 0 | 0 |
| 5% | 98.0725 | +0.9427 | | | -0.01 | +0.9359 |

IV. Immunization.

A. Dedication Strategies.

1. A dedication strategy ensures that a stream of liabilities can be met from available assets by holding a portfolio of fixed income assets whose cash flows exactly match the stream of fixed outflows.
2. Note that both the asset portfolio and the liability stream have the same current value and duration.
3. Example: It is 2/15/97 and QX must pay $5M on 8/15/97 and $10M on 2/15/98. A dedication strategy would involve buying Aug 97 U.S. strips with a face value of $5M and Feb 98 U.S. strips with a face value of $10M.

B. Target Date Immunization.

1. Assumptions.
   a. the yield curve is flat at y (APR with semiannual compounding).
   b. any shift in the curve keeps it flat.
2. Target date immunization ensures that a stream of fixed outflows can be met from available assets by holding a portfolio of fixed income assets:
   a. with the same current value as the liability stream; and,
   b. with the same modified duration.
3. Note,
   a. given the flat yield curve, all bonds have the same YTM, y.
   b. for this reason, a small shift in the yield curve will have the same effect on the value of the immunizing assets and on the value of the liabilities (using the definition of duration).
   c. and so, the assets will still be sufficient to meet the stream of fixed outflows.
   d. further, it is easy to work out the current value of the liability stream using y.
   e. equating the modified durations of the assets and the liabilities is the same as equating the durations.

4. Example: Firm GF is required to make a \$5M payment in 1 year and a \$4M payment in 3 years. The yield curve is flat at 10% APR with semi-annual compounding. Firm GF wants to form a portfolio using 1-year and 4-year U.S. strips to fund the payments. How much of each strip must the portfolio contain for it to still be able to fund the payments after a shift in the yield curve?
   a. The value of the liabilities is given by:

$$L(0) = 5M/[1+0.1/2]^2 + 4M/[1+0.1/2]^6 = 4.5351M + 2.9849M = 7.5200M.$$

   b. The duration of the liabilities is given by:

$$D^L(0) = 1 \times [4.5351/7.5200] + 3 \times [2.9849/7.5200] = 1.7938 \text{ years.}$$

   c. Let $A^1(0)$ be the portfolio's dollar investment in the 1-year strips and $A^4(0)$ be the portfolio's dollar investment in the 4-year strips.
   d. The dollar value of the portfolio must equal the value of the liabilities. So $A^1(0) + A^4(0) = 7.5200M$.
   e. The duration of the portfolio equals

$$D^P(0) = \omega^{1,P}\ D^1(0) + (1-\omega^{1,P})\ D^4(0)$$

   where $\omega^{1,P} = A^1(0)/ 7.5200M$. The duration of the 1-year strip is 1 and the duration of the 4-year strip is 4.
   f. Setting the duration of the portfolio equal to the duration of the liabilities gives:

$$1.7938 = \omega^{1,P}\ D^1(0) + (1-\omega^{1,P})\ D^4(0) = \omega^{1,P}\ 1 + (1-\omega^{1,P})\ 4 \Rightarrow \omega^{1,P} = 0.7354.$$

   g. Thus,

$$A^1(0) = 0.7354 \times 7.5200M = 5.5302M; \text{ and, } A^4(0) = 7.5200M - 5.5302M = 1.9898M.$$

5. Generalizations.
   a. The assumption of a flat yield curve has undesirable properties.
   b. When the yield curve is allowed to take more general shapes, target-date immunization is still possible, but modified Macaulay duration is not appropriate for measuring the impact of a yield curve shift on price. Need to use a more general duration measure.

C. Comparison.
   1. Dedication strategies are a particular type of target date immunization.
   2. The one advantage of a dedication strategy is that there is no need to rebalance through time. Almost all other immunization strategies involve reimmunizing over time.