**Lecture 15: Valuation Models.**

I. Reading.
   A. BKM, Chapter 18, except Section 18.6.

II. Introduction.
   A. Definition of Valuation.
      1. Valuation is the art/science of determining what a security or asset is worth.
      2. Sometimes we can observe a market value for a security and we are interested in assessing whether it is over or under valued (e.g., stock analysts); sometimes there is no market value and we are trying to construct one for bargaining or transaction purposes (e.e., a corporation is interested in selling a division.).
      3. The value of a security or asset is going to depend crucially on the asset pricing model we choose. (The effect is through the appropriate discount rate.)
      4. The most common kind of valuation problem is equity valuation.

III. Present Value Models.

A. General Approach.

1. These models assume that the stock is bought, held for some time (dividends are collected), and then sold.
2. The share is valued as the present value of the expected dividends and the expected proceeds from the sale. BKM call this the intrinsic value.

B. Example: Assume that dividends are paid annually and that the time 0 dividend has just been paid.

1. If the stock is held one year, the return on the stock is

$$R^i = \frac{D_1^i + P_1^i}{P_0^i} - 1$$

where $D_t^i$ is firm i's dividend per share at time t and $P_t^i$ is the stock price of the firm at t. Taking expectations and rearranging gives

$$P_0^i = \frac{E[D_1^i + P_1^i]}{1 + E[R^i]}$$

.

2. If the stock is held for two years, the present value is given by

$$P_0^i = \frac{E[D_1^i]}{1 + E[R^i]} + \frac{E[D_2^i + P_2^i]}{(1 + E[R^i])^2}.$$

C. If the stock is held until the company is liquidated, the present value is given by

$$P_0^i = \frac{E[D_1^i]}{1 + E[R^i]} + \frac{E[D_2^i]}{(1 + E[R^i])^2} + \ldots + \frac{E[D_\tau^i]}{(1 + E[R^i])^\tau} + \ldots$$

$$= \sum_{\tau=1}^{\infty} \frac{E[D_\tau^i]}{(1 + E[R^i])^\tau}$$

.

which is known as a dividend discount model (DDM).

D. Discussion.

1. The last formula highlights the relation between expected return and price and why we call a model that tells us something about expected return an asset pricing model.
2. We can see that holding expected dividends fixed, stock price today is decreasing in expected stock return; the higher the expected return needed to compensate for the stock's risk the lower the stocks price.

E. Two items affect the intrinsic value of an asset.

1. Expected Return.
2. Stream of Expected Cash Flows.

IV. Expected Return Determination.

A. Approaches:
   1. In a CAPM framework, use the SML; this approach allows you to explicitly make adjustments to your Beta estimate to reflect your assessment of the future Beta of the stock.
   2. If valuing existing equity, can also use a historical average return as an estimate of expected return.
   3. Can also adjust the estimate to take into account the predictability of returns and to allow for the sensitivity of the stock to other sources of risk (in an I-CAPM) setting; we will not focus on these adjustments here.

B. What determines equity Beta or equity risk loading.
   1. Can think of the firm as a portfolio of assets or a portfolio of claims on those assets:

$V = A^1 + A^2 + ... + A^J$

and

$V=S+B$

where
$V$ is the value of the firm;
$A^j$ is the value of the jth asset of the firm;
$S$ is the market value of the firm's equity;
$B$ is the market value of the firm's debt.

   2. Recall that Beta with respect to the market for a portfolio is a weighted average of the Betas of the assets that comprise the portfolio where the weights are the portfolio weights. In an I-CAPM context, the same is true for Beta with respect to other variables individuals care about.
   3. It follows that for Beta with respect to any variable (which of course includes Beta with respect to the market):

$$\beta_V = \frac{A^1}{V}\beta_{A1} + \frac{A^2}{V}\beta_{A2} + ... + \frac{A^J}{V}\beta_{AJ}.$$

and

$$\beta_V = \frac{S}{V}\beta_S + \frac{B}{V}\beta_B$$

where
$\beta_V$ is the Beta of the firm;
$\beta_{Aj}$ is the Beta of the jth asset of the firm;
$\beta_S$ is the Beta of the firm's equity;
$\beta_B$ is the Beta of the firm's debt.

4. Note
   a. If the firm's assets are unchanged, then firm Beta with respect to any variable is unchanged.
   b. Equity Beta can be calculated by rewriting the above formula:

$$\beta_S = \frac{V}{S}\beta_V - \frac{B}{S}\beta_B = \beta_V + \frac{B}{S}[\beta_V - \beta_B]$$

   c. Can see that equity Beta depends on:
      (1) the Betas of the firm's assets;
      (2) the level of debt of the firm; and
      (3) the Beta of the firm's debt.
   d. If the firm's debt is riskless, debt Beta with respect to any variable is 0 and so equity Beta can be calculated:

$$\beta_S = \frac{V}{S}\beta_V$$

C. Examples.
1. Suppose ZX company has a two assets. The first has a Beta with respect to the market of 1.5 while the second has a Beta with respect to the market of 0.9. The first asset is worth \$12M and the second is worth \$8M. The firm has \$4M of riskless debt. The CAPM holds for the economy, the riskless rate is 5% p.a. and the expected return on the market portfolio is 13% p.a. What is the expected return on ZX's equity?
   a. First, get the Beta of the firm:

$$\beta_{V,M} = \frac{A^1}{V}\beta_{A1,M} + \frac{A^2}{V}\beta_{A2,M} = \frac{12}{12+8}1.5 + \frac{8}{12+8}0.9 = 1.26.$$

   b. Second, get the Beta of the equity:

$$\beta_{S,M} = \frac{V}{S}\beta_{V,M} = \frac{20}{20-4}1.26 = 1.575.$$

   c. Third, use the SML to get the expected return on the equity:

$$E[R_S] = R_f + \{E[R_M] - R_f\}\beta_{S,M} = 5\% + \{13\% - 5\%\}\ 1.575 = 17.6\%.$$

2. IBM Example.
   a. CAPM
      (1) Inputs:
         (a) $E[R^M]$ based on the average monthly return on the S&P 500 for the period 12/90 to 12/95 is = $(1+0.01326)^{12} - 1 = 17.12\%$.
         (b) $R^f$ based on the 1-month T-bill yield at the start of 2/97 is = $(1+0.00323)^{12} - 1 = 3.95\%$.
         (c) $\beta_{IBM,M}$ can be obtained from the Bloomberg screen:
            i) 0.62 is the Beta obtained using monthly data from 12/90 to 12/95.
            ii) 0.75 is the corresponding adjusted Beta.
      (2) Using the SML:

$E[R^{IBM}]$ $= 3.95\% + 0.62 \times \{17.12\% - 3.95\%\} = 12.12\%$ using raw Beta.
$= 3.95\% + 0.75 \times \{17.12\% - 3.95\%\} = 13.83\%$ using adjusted Beta.

   b. Can also use an estimate of expected return based on historical average return.

V. Constant Growth DDM.

A. Model.

1. Suppose $E[D_1^i] = D_0^i \{1 + g^i\}$, $E[D_2^i] = E[D_1^i] \{1 + g^i\}$, ..., $E[D_{\tau+1}^i] = E[D_\tau^i] \{1 + g^i\}$.
2. So $g^i$ is the growth rate of the expected dividend which is assumed constant.
3. Can show that the DDM can be written:

$$P_0^i = \frac{D_0^i\{1 + g^i\}}{E[R^i] - g^i} = \frac{E[D_1^i]}{E[R^i] - g^i}$$ which is valid so long as $E[R^i] > g^i$.

B. IBM Example.

1. Aim is to value IBM stock as at 1/31/97.
2. Inputs.
   a. The total dividends paid in 1996 were \$0.25 + 3 \$0.35 = \$1.30 per share.
   b. Using the CAPM and raw Beta gives a discount rate of 12.12% p.a.
   c. The Earnings Estimates table indicates a growth rate in annual earnings over the next 5 years of 10.6%: this will be our estimate of $g^{IBM}$.
3. Using the constant growth DDM,

$P_0^{IBM} = D_0^{IBM} \{1 + g^{IBM}\} / \{E[R^{IBM}] - g^{IBM}\} = \$1.3x (1+0.106) / (0.1212-0.106) = 94.78.$

4. Compare this to the price of IBM at the end of January 1997 of 156.875. If we had full faith in our valuation we would consider IBM to be overvalued and issue a sell order.

C. Other Implications of the Constant Growth DDM.

1. Can rewrite the basic model:

$$E[R^i] = \frac{E[D_1^i]}{P_0^i} + g^i .$$

   a. This formula breaks required return into the expected dividend yield plus expected capital gain.
2. So $g^i$ is the expected capital gain on the stock (assuming no stock splits or stock dividends); can show this explicitly:

$$E[P_1^i] = \frac{E[D_1^i]\{1 + g^i\}}{E[R^i] - g^i} = P_0^i\{1 + g^i\}.$$

3. If we assume the stock is correctly valued, we can use the stock's dividend yield and earnings growth rate to calculate an estimate of expected return.

VI. Investment Opportunities.

A. Introduction.

1. Let $K_\tau^i$ be the book value of a share of equity at time $\tau$.
2. The book value per share evolves through time in the following way:

$$K_\tau^i = K_{\tau-1}^i + (E_\tau^i - D_\tau^i)$$

where $E_\tau^i$ are firm i's earnings (after interest) per share in period $\tau$. Any earnings not paid out as a dividend get added to the book value.

B. Assumptions.

1. The constant growth DDM correctly values stock.
2. Each year firm i's assets generate an expected after interest cash flow which is a constant fraction $ROE^i$ of the book value of the equity. This $ROE^i$ is known as the expected return on book equity. So the firm's expected earnings after interest for period $(\tau+1)$ are equal to the book value of the equity at the start of the period multiplied by $ROE^i$. In per share terms:

$$E[E_{\tau+1}^i] = K_\tau^i \ ROE^i.$$

3. Firm i pays a constant fraction $(1-b^i)$ of its earnings as a dividend. So $D_{\tau+1}^i = (1-b^i) E_{\tau+1}^i$ for any $\tau$.

   a. $(1-b^i)$ is called the payout ratio.
   b. $b^i$ is called the plowback or retention ratio.

4. IBM Example:
   a. Inputs.
      (1) Earnings per share for IBM for 1996 was \$10.24.
      (2) $D_0^{IBM} = \$1.3$.
   b. Can calculate $b^{IBM}$:

$(1-b^i) = D_0^i / E_0^i = 1.3/10.24 = 12.70\%$ and $b^i = 87.30\%$.

C. Implications.

1. Since dividend per share is a fixed fraction of earnings per share, it follows that expected earnings per share also grow at $g^i$:

$$\frac{E[E_1^i]}{E_0^i} = \frac{E[D_1^i]\{1 - b^i\}}{\{1 - b^i\}D_0^i} = \frac{E[D_1^i]}{D_0^i} = \{1 + g^i\}$$

2. IBM Example.

a. Taking the earnings growth estimate as our estimate of $g^{IBM}$ is consistent with this constant payout model.

b. Inputs:

(1) $g^{IBM} = 10.6\%$.

(2) $E_0^{IBM} = 10.24$.

c. Can calculate $E[E_1^{IBM}]$:

$E[E_1^{IBM}] = E_0^{IBM}\,[1+g^{IBM}] = 10.24\,[1+0.1060] = 11.33$.

3. What is the expected book value per share at time 1?

$E[K_1^i] = E[K_0^i + (E_1^i - D_1^i)] = K_0^i + K_0^i\ ROE^i - K_0^i\ ROE^i\,(1-b^i) = K_0^i\,(1 + ROE^i\ b^i)$.

Can show that the book value per share is also expected to grow at $g^i$:

$$\frac{E[K_1^i]}{K_0^i} = \frac{E[E_2^i]\ ROE^i}{ROE^i\ E[E_1^i]} = \frac{E[E_2^i]}{E[E_1^i]} = 1 + g^i.$$

Thus, have shown that $b^i\ ROE^i = g^i$ .

4. IBM Example.

a. Inputs:

(1) $b^{IBM} = 87.30\%$.

(2) $g^{IBM} = 10.6\%$.

(3) $E[E_1^{IBM}] = \$11.33$.

b. Can calculate $ROE^{IBM}$

$ROE^{IBM} = g^{IBM} / b^{IBM} = 0.106/0.8730 = 12.14\%$.

c. Can then calculate $K_0^{IBM}$ implied by the model:

$K_0^{IBM} = E[E_1^{IBM}] / ROE^{IBM} = \$11.33/0.1214 = \$93.33$

which can be compared with the actual book value at the end of 1996 of \$42.08. The difference between the two is a measure of the extent to which the assumptions about the evolution of book value hold for IBM.

D. Uses of the Model.

1. Valuation.
   a. Can easily show that the following formula must hold for the stock price of firm i:

$$P_0^i = \frac{D_0^i\{1 + g^i\}}{E[R^i] - g^i} = E_0^i\frac{\{1 + g^i\}\{1 - b^i\}}{E[R^i] - b^i ROE^i} = \frac{E[E_1^i]\{1 - b^i\}}{E[R^i] - b^i ROE^i}.$$

2. Optimal Plowback Ratio.
   a. If firm i paid out all its earnings as a dividend ($b^i$=0), its stock price at time zero would be $E[E_1^i]/E[R^i]$. The difference between this value and the constant growth DDM value is due to growth. Thus,

$$P_0^i = \frac{E[E_1^i]}{E[R^i]} + PVGO_0^i = \frac{E_0^i\{1 + g^i\}}{E[R^i]} + PVGO_0^i.$$

   where $PVGO_1^i$ is the value at time 0 of firm i's growth opportunities.
   b. Note that:
      (1) If $E[R^i] > ROE^i$:
         (a) $PVGO_1^i \leq 0$; and,
         (b) $b^i = 0$ maximizes $P_1^i$.
      (2) If $E[R^i] < ROE^i$:
         (a) $PVGO_1^i \geq 0$; and,
         (b) $P_1^i$ is increasing in $b^i$.
      (3) If $E[R^i] = ROE^i$:
         (a) $PVGO_1^i = 0$; and,
         (b) $P_1^i$ is unaffected by choice of $b^i$.
   c. IBM Example.
      (1) Inputs.
         (a) $P_0^{IBM} = \$94.78$.
         (b) $E[E_1^{IBM}] = \$11.33$.
         (c) $E[R^{IBM}] = 12.12\%$.
         (d) $ROE^{IBM} = 12.14\%$.
      (2) Can calculate $PVGO_0^{IBM}$:

$PVGO_0^{IBM} = P_0^{IBM} - \{E[E_1^{IBM}] / E[R^{IBM}]\} = \$94.78 - \$11.33/0.1212 = \$94.78 - \$93.49 = \$1.29.$

      (3) Note that $PVGO_0^{IBM} \geq 0$ as would be expected since $12.12\% = E[R^{IBM}] < ROE^{IBM} = 12.14\%$.

VII. Price/Earnings Approaches.

A. Definition.

1. The Price/Earnings or P/E ratio is defined as the price per share divided by the earnings per share (after interest).
2. IBM Example.
   a. As of the end of January 1997, the P/E ratio for IBM is given as 15.231 by Bloomberg. This can be obtained by dividing the price per share at the end of January 1997 by the earnings per share for 1996: 156.875/10.24 = 15.3.
3. The P/E ratio is sometimes used to describe the price as "IBM is selling at 15 times earnings".

B. Use of P/E.

1. The P/E ratio is sometimes used to get a rough measure of the intrinsic value of a company that is not publicly traded.
2. An average P/E ratio for all publicly traded firm in the industry is calculated.
3. The current earnings of the firm are multiplied by this average P/E to obtain an estimate of the firm's intrinsic value.

C. Caveat.

1. Indiscriminate use of the P/E ratio for valuation purposes can lead to trouble because of unstable accounting practices distorting accounting earnings.