Lecture 12: The Intertemporal CAPM (ICAPM): a Multifactor Model.

I. Reading.
   A. BKM, Chapter 10, Section 10.4.
   B. BKM, Chapter 27, Section 27.2.

II. ICAPM Assumptions.
   1. Same as CAPM except can not represent individual tastes and preferences in $\{E[R], \sigma[R]\}$ space.

III. When do individuals care about more than expected return and standard deviation?
   A. single period setting:
      1. returns are not normally distributed *and* individual utility depends on more than expected portfolio return and standard deviation.
   B. multiperiod setting:
      1. returns are not normally distributed *and* individual utility depends on more than expected portfolio return and standard deviation.
      2. individual preferences in the future depend on the state of the world at the end of this period;
      3. expected return and covariances of returns in future periods depends on the state of the world at the end of this period; e.g., predictable returns.

IV. Predictable Returns.

A. It has been empirically documented that expected stock returns over a period depend on variables known at the start of the period: e.g. dividend yield on the S&P 500 at the start of period t, DP(start t).

B. As described above (Lecture 5), the frame work for ascertaining this dependence is regression:

$$R_i(t) = \mu_{i,DP} + \varphi_{i,DP}\, DP(\text{start } t) + e_{i,DP}(t)$$

where DP is dividend yield on the S&P 500.

C. Let today be the start of February.

D. Expected return on asset i for February using DP at the start of February is given by :

$$E[R_i(\text{Feb})] = \mu_{i,DP} + \varphi_{i,DP}\, DP(\text{start Feb}).$$

E. Expected return on asset i for March using DP at the start of March is given by :

$$E[R_i(\text{Mar})] = \mu_{i,DP} + \varphi_{i,DP}\, DP(\text{end Feb}).$$

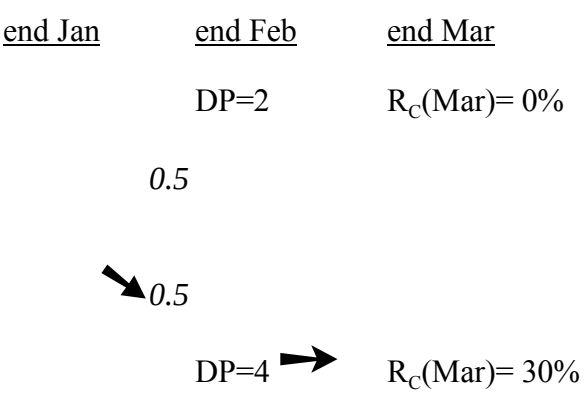
F. Empirically, find $\varphi_{i,DP} > 0$. So:

1. High DP(end Feb) is associated with high $E[R_i(\text{Mar})]$.
2. Low DP(end Feb) is associated with low $E[R_i(\text{Mar})]$.

G. So a high dividend yield at the end of this month implies high expected returns on stocks next month.

V. Example: Predictable Returns and a Two-period Perspective.

A. Two assets available at the start of February: A and B.
B. An individual can invest in either A or B but not both (assumed for simplicity).
C. Two states at the end of February: DP(end Feb) = 2 and DP(end Feb) = 4.
D. One riskless asset C available in March.
E. Individuals care about the portfolio's value at the end of March; i.e., care only about E[R(Feb-Mar)] and σ[R(Feb-Mar)].



Prob	DP(end Feb)	R_A(Feb)	R_B(Feb)	R_C(Mar)	R_{AC}(Feb-Mar)	R_{BC}(Feb-Mar)
0.5	2	20%	0%	0%	20%	0%
0.5	4	-10%	10%	30%	17%	43%

x=	DP(end Feb)	R_A(Feb)	R_{AC}(Feb-Mar)	R_B(Feb)	R_{BC}(Feb-Mar)
E[x]	3	5	18.5	5	21.5
σ[x]	1	15	1.5	5	21.5
σ[x,DP(end Feb)]		-15		5	

F. Single period Analysis: A vs B.

1. A and B have the same E[R(Feb)].
2. B has a smaller standard deviation.
3. So in a one-period setting in which investors only care about {σ[R(Feb)],E[R(Feb)]}, prefer B to A.

G. Two-period Analysis: A vs B.

1. Investors care about {σ[R(Feb-Mar)], E[R(Feb-Mar)]}.
2. R_C(Mar) high when DP(end Feb) high.
3. cov[R_A(Feb), DP(end Feb)]<0; so:
   a. R_A(Feb) high when R_C(Mar) low.
   b. R_A(Feb) low when R_C(Mar) high.
4. so R_{AC}(Feb-Mar) the return from investing in A then C is not very volatile: i.e., σ[R_{AC}(Feb-Mar)] is low.
5. cov[R_B(Feb), DP(end Feb)]>0; so:
   a. R_B(Feb) low when R_C(Mar) low.
   b. R_B(Feb) high when R_C(Mar) high.
6. so R_{BC}(Feb-Mar) the return from investing in B then C is very volatile: i.e., σ[R_{BC}(Feb-Mar)] is high.
7. thus, although E[R_{AC}(Feb-Mar)]<E[R_{BC}(Feb-Mar)], σ[R_{AC}(Feb-Mar)] is also less than σ[R_{BC}(Feb-Mar)].
8. whether an investor prefers A then C or B then C depends on how risk averse she is.
9. in a multiperiod setting with predictable returns, may prefer A to B.
10. what increases the attractiveness of A's February return is its negative covariance with DP(end Feb).
11. so when an investor forms her portfolio at the start of February, she cares about cov[R_p(Feb), DP(end Feb)] in addition to E[R_p(Feb)] and σ[R_p(Feb)].

VI. Tastes and Preferences with Predictable Returns and a Long-term Investment Horizon.

A. Intuition of previous example carries over to more general settings in which
   1. individuals live for more than 2 months.
   2. there are more than two states of the world.
   3. more than one macro-economic variable predicts asset returns.

B. So, in general, if asset returns over period t+1 are predictable using DP(end t) then can only fully represent an individual's tastes and preferences for her period t portfolio return using $\{E[R(t)], \sigma[R(t)], \sigma[R(t), DP(\text{end } t)]\}$.

C. Even more generally, if asset returns over period t+1 are predictable using K state variables, $s_1(\text{end } t)$, ..., $s_K(\text{end } t)$ then can only fully represent an individual's tastes and preferences for her period t portfolio return using $E[R(t)]$, $\sigma[R(t)]$, $\sigma[R(t), s_1(\text{end } t)]$, ..., $\sigma[R(t), s_K(\text{end } t)]\}$.

VII. Portfolio Choice.

A. Since individual's care about more than expected return and standard deviation of return, individuals no longer choose mean variance efficient portfolios (i.e., portfolios on the positive sloped portion of the MVF for all assets).

B. Example: Predictable Returns
   1. Today is the start of February.
   2. A portfolio on the MVF may have a high covariance with DP at the end of February.
   3. Thus, an individual may prefer to hold another portfolio in February not on the MVF but which has a very low covariance with DP at the end of February.

VIII. Individual Assets.

A. Since individuals do not necessarily hold portfolios on the positive sloped part of the MVF, the market need not lie on the positive sloped part of the MVF.

B. Minimum variance mathematics then tells us that there need not be a linear relation between expected return and Beta with respect to the market portfolio; i.e., assets need not all lie on the SML:

$$E[R_i] \neq R_f + \beta_{i,M} \{E[R_M] - R_f\}.$$

C. Instead, if individuals care about the covariance of portfolio return with a set of state variables $s_1, s_2, \ldots, s_K$, returns and the state variables are multivariate normally distributed then can show that the following holds for all assets and portfolios of assets:

$$E[R_i] = R_f + \beta_{i,M} \lambda_M + \beta_{i,s1} \lambda_1 + \beta_{i,s2} \lambda_2 + \ldots + \beta_{i,sK} \lambda_K$$

where:

$\lambda_M, \lambda_1, \lambda_2, \ldots, \lambda_K$ are constants that are the same for all assets ; and
$\beta_{i,sk} = cov[R_i, s_k]/var[s_k]$ for k=1,2,...,K; and,
$\beta_{i,M} = cov[R_i, R_M]/var[R_M]$ as previously defined.

D. Note: $\lambda_M \neq E[R_M]-R_f$ since $\beta_{M,M} = 1$ but $\beta_{M,s1} \neq 0$, $\beta_{M.s2} \neq 0$, ... , $\beta_{M,sK} \neq 0$.

E. Example (cont): If individuals care about covariance of portfolio return over t with DP(end t) and asset returns over t and DP(end t) are multivariate normally distributed, the following holds for all assets:

$$E[R_i] = R_f + \beta_{i,M} \lambda_M + \beta_{i,DP} \lambda_{DP}$$

where:

λ_M, λ_{DP} are constants that are the same for all assets and portfolios.
$\beta_{i,DP} = cov[R_i(t), DP(end\ t)]/var[DP(end\ t)]$.

1. Note:

a. Slope of the predictive regression for $R_i(t)$ using DP(start t) as the predictor is

$$\varphi_{i,DP} = cov[R_i(t), DP(start\ t)]/var[DP(start\ t)].$$

b. So $\beta_{i,DP}$ is obtained by regressing $R_i(t)$ on the S&P 500 dividend yield at the *end* of t while the slope of the predictive regression requires $R_i(t)$ to be regressed on the S&P 500 dividend yield at the *start* of t.

F. More usually, see the result expressed in terms of regression coefficients from a regression of portfolio return on the market return and the K state variables:

$$E[R_i] = R_f + \beta^*_{i,M} \lambda^*_M + \beta^*_{i,s1} \lambda^*_1 + \beta^*_{i,s2} \lambda^*_2 + ... + \beta^*_{i,sK} \lambda^*_K$$

where:

$\lambda^*_M, \lambda^*_1, \lambda^*_2,..., \lambda^*_K$ are constants that are the same for all assets and portfolios; and

$\beta^*_{i,sk}$ for k=1,2,...,K, and $\beta^*_{i,M}$ are regression coefficients from a multivariate regression of R_i on R_M, s_1, s_2, ... and s_K:

$$R_i = \varphi_{i,0} + \beta^*_{i,M} R_M + \beta^*_{i,s1} s_1 + \beta^*_{i,s2} s_2 + ... + \beta^*_{i,sK} s_K + e_i$$

G. Note:

1. $\beta^*_{i,sk}$ for k=1,2,...,K, and $\beta^*_{i,M}$ are referred to as risk loadings and vary across assets; they measure the sensitivity of asset i to each of the risks that individuals care about.
2. $\lambda^*_M, \lambda^*_1, \lambda^*_2,..., \lambda^*_K$ are referred to as risk premia and measure the expected return compensation an investor must receive to bear one unit of the relevant risk.
3. $\lambda^*_M = E[R_M]-R_f$ since when R_M is regressed on R_M, s_1, s_2, ... and s_K get $\beta^*_{M,M} = 1$ and $\beta^*_{M,s1} = \beta^*_{M,s2} = ... = \beta^*_{M,sK} = 0$.

IX. CAPM vs ICAPM

A. It can easily be seen that the CAPM is a special case of this ICAPM model.

B. In particular, the expressions for expected return on any asset in VIII.C and VIII.F above, reduce to the CAPM when K=0; i.e., when investors only care about E[R] and σ[R].

X. Numerical Example. Let DEF(Jan) be the difference in the yield on a long term low-grade corporate bond and the yield on a long term government bond at the end of January and let R_M(Jan) be the January return on the market portfolio. Suppose each individual cares about $\{E[R_p(Jan)], \sigma[R_p(Jan)], \sigma[R_p(Jan), DEF(Jan)]\}$ when forming his/her portfolio p for January. The following additional information is available:

i	$E[R_i(Jan)]$	$\beta_{i,M}$	$\beta_{i,DEF}$
Pink	1.73%	1.3	0.25
Grey	1.34%	0.9	0.10
Black	?	0.9	0.05

where $\beta_{i,M}$ is the regression coefficients from a regression (time-series) of $R_i(t)$ on $R_M(t)$:

$$R_i(t) = \varphi_{i,0} + \beta_{i,M} R_M(t) + e_i(t),$$

and $\beta_{i,DEF}$ is the regression coefficients from a regression (time-series) of $R_i(t)$ on DEF(t):

$$R_i(t) = \varphi_{i,0} + \beta_{i,DEF} DEF(t) + u_i(t).$$

Also know that riskless rate for January, R_f(Jan), 0.7% and that $\beta_{M,DEF}$ is 0.20.

1. What is the expected January return on Black?

Know ICAPM holds. So all assets lie on

$$E[R_i(Jan)] = R_f(Jan) + \beta_{i,M} \lambda_M + \beta_{i,DEF} \lambda_{DEF} .$$

Using this formula for Pink and Grey:

Pink: $1.73 = 0.7 + 1.3 \lambda_M + 0.25 \lambda_{DEF}$

Grey: $1.34 = 0.7 + 0.9 \lambda_M + 0.10 \lambda_{DEF}$

Now Pink $\Rightarrow \lambda_{DEF} = 4 (1.03\% - 1.3 \lambda_M)$

which can be substituted into Grey to obtain

$1.34 = 0.7 + 0.9 \lambda_M + 0.10 \times 4 (1.03\% - 1.3 \lambda_M)$.

It follows that $\lambda_M = 0.6\%$ and $\lambda_{DEF} = 1\%$.

Know that Black satisfies:

$$E[R_{Black}(Jan)] = R_f(Jan) + \beta_{Black,M} \lambda_M + \beta_{Black,DEF} \lambda_{DEF}$$

$= 0.7 + \beta_{Black,M}\, 0.6 + \beta_{Black,DEF}\, 1$
$= 0.7 + 0.9 \times 0.6 + 0.05 \times 1$
$= 1.29\%$

which is not the same as the expected return for Grey.

2. What is the expected January return on the market portfolio $E[R_M(Jan)]$?

$E[R_M(Jan)] = R_f(Jan) + \beta_{M,M}\, \lambda_M + \beta_{M,DEF}.\, \lambda_{DEF}$
$= 0.7 + \beta_{M,M}\, 0.6 + \beta_{M,DEF}\, 1$
$= 0.7 + 1 \times 0.6 + 0.2 \times 1$
$= 1.5\% \neq \lambda_M + R_f(Jan) = 0.6 + 0.7 = 1.3\%$

3. Is the market portfolio on the minimum variance frontier of the risky assets in the economy? Why or why not?

No. The reason is that individual's care about more than just $E[R]$ and $\sigma[R]$.