King Fahd University of Petroleum & Minerals
Department of Mathematics & Statistics
Math 513 Major Exam 1
The First Semester of 2026-2027 (261)
Time Allowed: 120 Minutes

Name: _____ ID#: _____

Instructor: _____ Sec #: _____ Serial #: _____

- Mobiles and calculators are not allowed in this exam.
- Write all steps clear.

Question #	Marks	Maximum Marks
1		12
2		16
3		12
4		12
5		16
6		12
Total		80

Q:1 (12 points) Solve the homogeneous linear system using matrix exponential method,

$$x'(t) = 5x - 6y$$
$$y'(t) = 2x - 2y$$

under the initial condition $\begin{bmatrix} x(0) \\ \\ y(0) \end{bmatrix} = \begin{bmatrix} 1 \\ \\ 2 \end{bmatrix}$. Write the solutions $x(t)$ and $y(t)$ of the linear system.

Q:2 (16 points) Matrix exponential of the coefficient matrix of the linear system

$$x'(t) = 2x - y + e^t$$
$$y'(t) = 3x - 2y + 1$$

is $e^{At} = \begin{bmatrix} -\frac{1}{2}e^{-t} + \frac{3}{2}e^{t} & \frac{1}{2}e^{-t} - \frac{1}{2}e^{t} \\ -\frac{3}{2}e^{-t} + \frac{3}{2}e^{t} & \frac{3}{2}e^{-t} - \frac{1}{2}e^{t} \end{bmatrix}$

Find the general solutions $x(t)$ and $y(t)$ of the nonhomogeneous linear system under the initial condition $\begin{bmatrix} x(0) \\ y(0) \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$.

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Q:3 (12 points) Find the Fourier series of $f(t) = \begin{cases} t & -\pi < t \leq 0, \\ 0 & 0 \leq t < \pi. \end{cases}$

Q:4 (12 points) Find the complex Fourier series of $f(t) = \cos(t)$ for $0 \leq t \leq \pi$.

Q:5 (16 points) Solve the differential equation $y'' - y = f(t)$, where $f(t) = \dfrac{\pi - t}{2}, \quad 0 < t < 2.$ and $f(\tau) = f(\tau + 2)$. Use Fourier series $f(t) = \dfrac{\pi - 1}{2} + \dfrac{1}{\pi} \displaystyle\sum_{n=1}^{\infty} \frac{\sin(n\pi t)}{n}$.

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Q:6 (12 points) Find eigenvalues and eigenfunctions of the Sturm-Liouville problem,

$$y'' + \lambda y = 0, \quad y'(0) = 0, \quad y'(\pi) = 0.$$