Dept of Mathematics and Statistics
King Fahd University of Petroleum & Minerals
AS380: Actuarial Contingencies I
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Midterm Exam Term 261
Tuesday, October 13, 2026
3:00 PM - 4:40 PM

Name: _____ **ID#:** _____

Instructions:

1. Please turn off your cell phones and place them under your chair. Any student caught with mobile phones on during the exam will be considered under the cheating rules of the University.
2. If you need to leave the exam hall, you are not allowed to do so until you have finished your exam. You may leave the hall before the start of the exam if you need to leave at all. No extra time will be provided for the time spent outside the room.
3. Only materials provided by the instructor can be present on the table during the exam.
4. Do not spend too much time on any one question. If a question seems too difficult, leave it and go on.
5. Use the blank portions of each page for your work. Extra blank pages can be provided if necessary. If you use an extra page, indicate clearly what problem you are working on.
6. Only answers supported by work will be considered. Unsupported guesses will not be graded.
7. While every attempt is made to avoid defective questions, sometimes they do occur. In the rare event that you believe a question is defective, the instructor cannot give you any guidance beyond these instructions.
8. **Only answers supported by work will be considered. Unsupported guesses will not be graded.**
9. Mobile calculators, I-pad, or communicable devices are disallowed. Use regular scientific calculators, financial calculators, or SOA-approved calculators only. *Write important steps to arrive at the solution of the exam problems.*

The test is 100 minutes, GOOD LUCK, and you may begin now!

Question	Total Point	Point Obtained	Comments
A1	5		
A2	6		
B1	3		
B2	3		
B3	3		
Total	**20**		

Part A: Multiple Choice Questions

Part A1: Concept Questions (5 points)

1. Which of the following best describes the concept of "underwriting" in life insurance?

   (A) The process of investing premiums to earn returns.

   (B) The process of evaluating risks to classify policyholders into homogeneous groups.

   (C) The process of calculating the reserves required to meet future liabilities.

   (D) The process of distributing profits to policyholders.

   (E) The process of determining the premium rates for a new policy.

2. A life table is a fundamental tool in actuarial science. Which of the following statements is **TRUE** regarding a life table?

   (A) The radix of a life table must be equal to 1.

   (B) The function l_x represents the probability of survival from birth to age x.

   (C) The function d_x represents the expected number of deaths between ages x and $x+1$.

   (D) The life table is only valid for a population with a constant force of mortality.

   (E) The function q_x represents the number of lives surviving to age $x+1$.

3. Under the Uniform Distribution of Deaths (UDD) assumption, which of the following is **TRUE** for an integer age x and $0 \leq t < 1$?

   (A) The force of mortality is constant between integer ages.

   (B) The probability of death is uniformly distributed over the year of age.

   (C) The survival function is a non-linear function of t.

   (D) The random variable T_x is uniformly distributed over the year of age.

   (E) The expected kurtate lifetime is exactly $e_x + 0.5$.

4. Which of the following types of insurance is designed to provide financial protection to beneficiaries upon the death of the insured?

   (A) Health Insurance

   (B) Property Insurance

   (C) Life Insurance

   (D) Auto Insurance

   (E) Liability Insurance

5. Which of the following is a key difference between a life annuity and an annuity-certain?

   (A) A life annuity has a fixed number of payments, while an annuity-certain does not.

   (B) The present value of a life annuity is always higher than that of an equivalent annuity-certain.

   (C) A life annuity is contingent on survival, while an annuity-certain is not.

   (D) A life annuity is only used in pension plans, while an annuity-certain is used for loans.

   (E) A life annuity cannot be valued using actuarial notation.

6. What is the primary role of an actuary in the insurance industry?

   (A) To sell insurance policies to customers.

   (B) To manage the investment portfolio of the insurance company.

   (C) To evaluate and quantify risks, and design insurance products.

   (D) To handle customer service and claims processing.

   (E) To ensure the company complies with tax laws.

7. Which of the following statements best describes the difference between the Uniform Distribution of Deaths (UDD) assumption and the Constant Force of Mortality assumption?

   (A) UDD assumes the force of mortality is constant between integer ages, while the constant force assumption assumes deaths are uniformly distributed.

   (B) UDD assumes deaths are uniformly distributed over each year of age, while the constant force assumption assumes the force of mortality is constant between integer ages.

   (C) UDD is used for select models, while the constant force assumption is used for ultimate models.

   (D) UDD assumes the survival function is linear between integer ages, while the constant force assumption assumes the survival function is exponential.

   (E) Both assumptions are mathematically equivalent and produce identical results for all fractional ages.

8. Which of the following is **NOT** a valid condition for a survival function $S_x(t)$?

   (A) $S_x(0) = 1$

   (B) $\lim_{t\to\infty} S_x(t) = 0$

   (C) $S_x(t)$ must be a non-increasing function of t.

   (D) $S_x(t)$ must be a linear function of t.

   (E) $S_x(t)$ must be differentiable for all $t > 0$.

9. In the context of life insurance, what does the term "endowment insurance" refer to?

   (A) An insurance policy that only pays a benefit on death.

   (B) An insurance policy that only pays a benefit on survival to a certain date.

   (C) An insurance policy that pays a benefit on either death or survival to a certain date, whichever occurs first.

   (D) An insurance policy that provides a series of payments after retirement.

   (E) An insurance policy that covers only accidental death.

10. Which of the following statements about the force of mortality, μ_x, is **TRUE**?

    (A) It is the probability that a life aged x dies within the next year.

    (B) It is always a decreasing function of age x.

    (C) It represents the instantaneous rate of mortality at age x.

    (D) It is only defined for integer ages.

    (E) It is independent of the survival function $S_x(t)$.

Part A2: Calculation Questions (6 points)

11. You are given that $p_{60} = 0.98$, $p_{61} = 0.975$, and $q_{62} = 0.03$. Calculate ${}_3p_{60}$.

    (A) 0.9124

    (B) 0.9269

    (C) 0.9555

    (D) 0.9700

    (E) 0.9412

12. Given the following life table extract:

x	l_x
50	1000
51	980
52	950

    Calculate q_{51} assuming a uniform distribution of deaths between integer ages.

    (A) 0.0200

    (B) 0.0306

    (C) 0.0316

    (D) 0.0408

    (E) 0.0500

13. A life aged 40 purchases a 10-year pure endowment policy with a sum insured of \$50,000. The benefit is payable if the policyholder survives to age 50. Assuming an interest rate of 5% per year and ${}_{10}p_{40} = 0.95$, calculate the EPV of this benefit.

    (A) \$23,750.00

    (B) \$29,158.50

    (C) \$30,695.25

    (D) \$31,250.00

    (E) \$47,500.00

14. Under an endowment insurance issued to a life aged x, let X denote the present value of a unit sum insured, payable at the moment of death or at the end of the n-year term. Under a term insurance issued to a life aged x, let Y denote the present value of a unit sum insured, payable at the moment of death within the n-year term. You are given:

$$\mathrm{V}[X] = 0.0052, \quad v^n = 0.3, \quad {}_np_x = 0.8, \quad \mathrm{E}[Y] = 0.04.$$

    Calculate $\mathrm{V}[Y]$.

    (A) 0.0025

    (B) 0.0050

    (C) 0.0075

    (D) 0.0100

    (E) 0.0125

...

15. An insurer issues a whole life insurance policy to a life aged 65. The sum insured is \$100,000, payable at the end of the quarter-year of death. You are given that $A_{65}^{(4)} = 0.45$ and $i = 0.04$. Calculate the net single premium for this policy.

    (A) \$40,000

    (B) \$43,269

    (C) \$45,000

    (D) \$46,875

    (E) \$55,000

16. A life aged 60 purchases a whole life annuity-due of \$10,000 per year. You are given that $\ddot{a}_{60} = 12.5$. Calculate the EPV of this annuity.

    (A) \$100,000

    (B) \$120,000

    (C) \$125,000

    (D) \$150,000

    (E) \$200,000

Part B: Written Questions (9 points)

Instructions: Answer all questions. Show all your work.

Question B1. Let

$$F_0(t) = 1 - \left(1 - \frac{t}{105}\right)^{1/5} \quad \text{for } 0 \leq t \leq 105$$

(i) Calculate the probability that a new born life dies before age 60.

(ii) The probability that a life age 30 survives to at least age 70.

(iii) The probability that a life age 20 dies between ages 90 and 100.

(iv) The force of mortality at age 50.

(v) The median future lifetime at age 50.

(vi) The complete expectation of life at age 50.

Question B2. You are given the following life table extract:

Age, x	l_x
52	89,948
53	89,089
54	88,176
55	87,208
56	86,181
57	85,093
58	83,940
59	82,719
60	81,429

Calculate:

(a) ${}_{0.2}q_{52.4}$ assuming UDD (fractional age assumption).

(b) ${}_{0.2}q_{52.4}$ assuming constant force of mortality (fractional age assumption).

(c) ${}_{5.7}q_{52.4}$ assuming UDD.

(d) ${}_{5.7}q_{52.4}$ assuming constant force of mortality.

(e) ${}_{3.2|2.5}q_{52.4}$ assuming UDD.

(f) ${}_{3.2|2.5}q_{52.4}$ assuming constant force of mortality.

Question B3. For three insurance policies on the same life, you are given:

(i) Z_1 is the present value of a 20-year term insurance with sum insured \$200, payable at the end of the year of death, with $\mathrm{E}[Z_1] = 6.6$ and $\mathrm{V}[Z_1] = 748$.

(ii) Z_2 is the present value of a 20-year deferred whole life insurance with sum insured \$1000, payable at the end of the year of death, with $\mathrm{E}[Z_2] = 107.5$ and $\mathrm{V}[Z_2] = 5078$.

(iii) Z_3 is the present value of a whole life insurance with sum insured \$2000, payable at the end of the year of death.

Calculate the mean and standard deviation of Z_3.

Formula Sheet

Standard Ultimate Survival Model: $\mu_x = A + Bc^x$ with $A = 0.00022, B = 2.7 \times 10^{-6}, c = 1.124$. **Interest Theory:** $v = \frac{1}{1+i}$, $\delta = \ln(1+i)$, $d = 1-v$, $i^{(m)} = m((1+i)^{1/m} - 1)$. **Survival Models:** ${}_tp_x = \frac{l_{x+t}}{l_x}$, ${}_tq_x = 1 - {}_tp_x$, ${}_{u|t}q_x = {}_up_x - {}_{u+t}p_x$. **Force of Mortality:** $\mu_x = \frac{f_0(x)}{S_0(x)}$, ${}_tp_x = \exp\left(-\int_0^t \mu_{x+s} ds\right)$. **Fractional Age Assumptions:**

- UDD: ${}_sq_x = sq_x$ for $0 \leq s < 1$.
- Constant Force: ${}_sp_x = (p_x)^s$ for $0 \leq s < 1$.

Insurance Benefits (EPV):

- Whole Life (continuous): $\bar{A}_x = \int_0^\infty e^{-\delta t} {}_tp_x \mu_{x+t} dt$
- Whole Life (annual): $A_x = \sum_{k=0}^\infty v^{k+1} {}_k|q_x$
- Term (annual): $A^1_{x:n} = \sum_{k=0}^{n-1} v^{k+1} {}_k|q_x$
- Pure Endowment: ${}_nE_x = v^n {}_np_x$
- Endowment: $A_{x:n} = A^1_{x:n} + {}_nE_x$

Annuity Benefits (EPV):

- Whole Life Annuity-Due: $\ddot{a}_x = \sum_{k=0}^\infty v^k {}_kp_x = \frac{1-A_x}{d}$
- Term Annuity-Due: $\ddot{a}_{x:n} = \sum_{k=0}^{n-1} v^k {}_kp_x = \frac{1-A_{x:n}}{d}$
- Whole Life Immediate: $a_x = \ddot{a}_x - 1$

Recursions: $A_x = vq_x + vp_x A_{x+1}$, $\ddot{a}_x = 1 + vp_x \ddot{a}_{x+1}$. **Approximations:**

- UDD: $\bar{A}_x \approx \frac{i}{\delta} A_x$, $A_x^{(m)} \approx \frac{i}{i^{(m)}} A_x$
- Claims Acceleration: $\bar{A}_x \approx (1+i)^{1/2} A_x$, $A_x^{(m)} \approx (1+i)^{\frac{m-1}{2m}} A_x$

Woolhouse's Formula (3-term): $\ddot{a}_x^{(m)} \approx \ddot{a}_x - \frac{m-1}{2m} - \frac{m^2-1}{12m^2}(\delta + \mu_x)$. **Additional Formulas:**

- Variance of whole life insurance (continuous): $V(\bar{Z}_x) = {}^2\bar{A}_x - (\bar{A}_x)^2$
- Variance of whole life insurance (annual): $V(Z_x) = {}^2A_x - (A_x)^2$
- Variance of whole life annuity-due: $V(\ddot{a}_{K_x+1}) = \frac{{}^2A_x - (A_x)^2}{d^2}$
- Constant Force of Mortality: ${}_tp_x = e^{-\mu t}$, $\bar{A}_x = \frac{\mu}{\mu+\delta}$

Answer Key and Brief Solutions

Part A: Multiple Choice Questions

Section A1: Concept Questions

1. **B**

Underwriting is the process of assessing risk to classify individuals into groups with similar risk profiles.

2. **C**

$d_x = l_x - l_{x+1}$ is the expected number of deaths between ages x and $x+1$.

3. **B**

UDD assumes deaths are uniformly distributed over each year of age, so ${}_sq_x = sq_x$.

4. **C**

Life insurance provides financial protection to beneficiaries upon the death of the insured.

5. **C**

A life annuity is contingent on survival, whereas an annuity-certain has fixed payments.

6. **C**

Actuaries evaluate and quantify risks, and design insurance products.

7. **B**

UDD assumes that the probability of death is uniformly distributed over each year of age, meaning ${}_sq_x = sq_x$ for $0 \leq s < 1$. The constant force of mortality assumption assumes that the force of mortality, μ_{x+s}, is constant for $0 \leq s < 1$, which implies that ${}_sp_x = (p_x)^s$. Option B correctly identifies both definitions.

8. **D**

Survival functions do not need to be linear. They must be non-increasing, start at 1, and approach 0.

9. **C**

Endowment insurance pays a benefit on death or on survival to the end of the term, whichever occurs first.

10. **C**

The force of mortality μ_x is the instantaneous rate of mortality at age x.

Section A2: Calculation Questions

11. **B**

Source: Exercise 2.1
$p_{60} = 0.98$, $p_{61} = 0.975$, $q_{62} = 0.03 \Rightarrow p_{62} = 0.97$.
${}_3p_{60} = p_{60} \cdot p_{61} \cdot p_{62} = 0.98 \times 0.975 \times 0.97 = 0.9269$.

12. **B**

Source: Exercise 3.2
$q_{51} = \frac{l_{51} - l_{52}}{l_{51}} = \frac{980 - 950}{980} = \frac{30}{980} \approx 0.0306$.

13. **B**

Source: Section 4.4.6
$\text{EPV} = 50000 \cdot v^{10} \cdot {}_{10}p_{40} = 50000 \cdot (1.05)^{-10} \cdot 0.95 = 50000 \cdot 0.6139 \cdot 0.95 = 29158.50$.

14. **D**

Source: Exercise 4.6
Let $Z = X - Y$ be the present value random variable for a pure endowment. Then $Z = v^n$ if $T_x > n$, and 0 otherwise.
$\text{E}[Z] = {}_nE_x = v^n {}_np_x = 0.3 \times 0.8 = 0.24$.
$\text{E}[Z^2] = v^{2n} {}_np_x = (0.3)^2 \times 0.8 = 0.072$.
$\text{V}[Z] = \text{E}[Z^2] - (\text{E}[Z])^2 = 0.072 - (0.24)^2 = 0.0144$.
$\text{E}[X] = \text{E}[Y] + \text{E}[Z] = 0.04 + 0.24 = 0.28$.
Since $Y = X - Z$, we have $\text{V}[Y] = \text{V}[X] + \text{V}[Z] - 2\text{Cov}[X, Z]$.
$\text{Cov}[X, Z] = \text{E}[XZ] - \text{E}[X]\text{E}[Z]$. Note that $XZ = Z^2$ (since Z is either 0 or v^n), so $\text{E}[XZ] = \text{E}[Z^2] = 0.072$.
$\text{Cov}[X, Z] = 0.072 - 0.28 \times 0.24 = 0.0048$.
$\text{V}[Y] = 0.0052 + 0.0144 - 2(0.0048) = 0.0100$.

15. **C**

Source: Section 4.4.2
Net single premium $= 100000 \cdot A_{65}^{(4)} = 100000 \cdot 0.45 = 45000$.

16. **C**

Source: Example 5.1
$\text{EPV} = 10000 \cdot \ddot{a}_{60} = 10000 \cdot 12.5 = 125000$.

Part B: Written Questions

1. *Source: Exercise 2.6*
(i) $F_0(60) = 1 - (1 - 60/105)^{1/5} = 1 - (45/105)^{1/5} = 1 - (0.4286)^{0.2} \approx 0.1559$.
(ii) ${}_{40}p_{30} = \frac{S_0(70)}{S_0(30)} = \frac{(1-70/105)^{1/5}}{(1-30/105)^{1/5}} = \frac{(35/105)^{1/5}}{(75/105)^{1/5}} = \frac{(0.3333)^{0.2}}{(0.7143)^{0.2}} \approx 0.8586$.
(iii) ${}_{70|10}q_{20} = \frac{S_0(90) - S_0(100)}{S_0(20)} = \frac{(15/105)^{1/5} - (5/105)^{1/5}}{(85/105)^{1/5}} \approx 0.1394$.
(iv) $\mu_{50} = \frac{-1}{S_0(50)} \frac{d}{dx} S_0(50) = \frac{1}{525 - 5(50)} = \frac{1}{275} \approx 0.0036$.
(v) Median: Solve $S_{50}(t) = 0.5 \Rightarrow \left(1 - \frac{t}{55}\right)^{1/5} = 0.5 \Rightarrow t = 55(1 - 0.5^5) \approx 53.28$.
(vi) $\mathring{e}_{50} = \int_0^{55} {}_tp_{50} dt = \int_0^{55} \left(1 - \frac{t}{55}\right)^{1/5} dt = 55 \times \frac{5}{6} = 45.83$.

2. *Source: Exercise 3.2*
   (a) ${}_{0.2}q_{52.4} = 1 - \frac{l_{52.6}}{l_{52.4}} = 1 - \frac{0.4l_{52}+0.6l_{53}}{0.6l_{52}+0.4l_{53}} = 0.001917.$
   (b) ${}_{0.2}q_{52.4} = 1 - (p_{52})^{0.2} = 1 - \left(\frac{89089}{89948}\right)^{0.2} = 0.001917.$
   (c) ${}_{5.7}q_{52.4} = 1 - \frac{l_{58.1}}{l_{52.4}} = 1 - \frac{0.9l_{58}+0.1l_{59}}{0.6l_{52}+0.4l_{53}} = 0.064578.$
   (d) ${}_{5.7}q_{52.4} = 1 - (p_{52})^{0.6} \cdot {}_5p_{53} \cdot (p_{58})^{0.1} = 0.064577.$
   (e) ${}_{3.2|2.5}q_{52.4} = \frac{l_{55.6}-l_{58.1}}{l_{52.4}} = 0.030957.$
   (f) ${}_{3.2|2.5}q_{52.4} = 0.030950.$

3. *Source: Exercise 4.25*
   Let Z denote the present value of a whole life insurance with sum insured \$1, payable at the end of the year of death. Then:

   - $Z_1 = 200 \cdot Z \cdot I(T_x \leq 20)$
   - $Z_2 = 1000 \cdot Z \cdot I(T_x > 20)$
   - $Z_3 = 2000 \cdot Z$

   The key is to express Z_3 in terms of Z_1 and Z_2. Note that:

$$Z_3 = 2000 \cdot Z = 2000 \cdot Z \cdot [I(T_x \leq 20) + I(T_x > 20)]$$

$$Z_3 = \frac{2000}{200} Z_1 + \frac{2000}{1000} Z_2 = 10Z_1 + 2Z_2$$

   Mean:

$$\mathrm{E}[Z_3] = 10\mathrm{E}[Z_1] + 2\mathrm{E}[Z_2] = 10(6.6) + 2(107.5) = 66 + 215 = 281$$

   Variance: Since Z_1 and Z_2 are mutually exclusive (one applies if death occurs within 20 years, the other if death occurs after 20 years), they cannot both be non-zero simultaneously. Therefore, $Z_1 Z_2 = 0$, which implies:

$$\mathrm{Cov}[Z_1, Z_2] = \mathrm{E}[Z_1 Z_2] - \mathrm{E}[Z_1]\mathrm{E}[Z_2] = 0 - (6.6)(107.5) = -709.5$$

   Now, using the variance formula for a linear combination:

$$\mathrm{V}[Z_3] = \mathrm{V}[10Z_1 + 2Z_2] = 100\mathrm{V}[Z_1] + 4\mathrm{V}[Z_2] + 2(10)(2)\mathrm{Cov}[Z_1, Z_2]$$

$$\mathrm{V}[Z_3] = 100(748) + 4(5078) + 40(-709.5)$$

$$\mathrm{V}[Z_3] = 74800 + 20312 - 28380 = 66732$$

$$\mathrm{SD}[Z_3] = \sqrt{66732} \approx 258.33$$

Mean = 281, Standard Deviation = 258.33